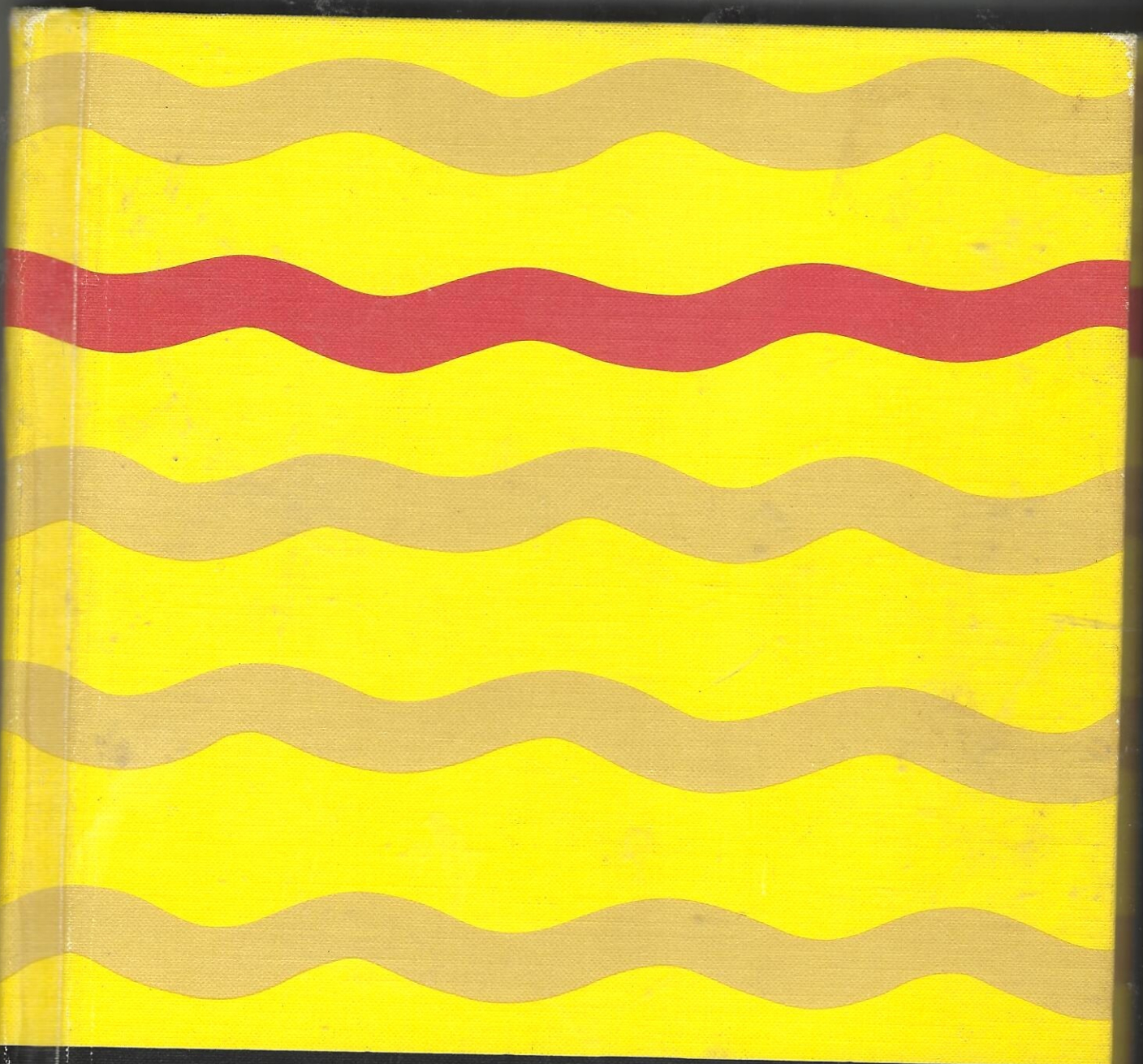

Arithmetic in My World

Arithmetic

IN MY WORLD

by

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6

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6

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The authors believe that the learning and teaching of arithmetic is based on child growth. Through the study of many thousands of children and their problems, they have determined how and what children should learn in order to establish desirable growth patterns. They have found that children's problems can be organized into units of work composed of activities which are in reality areas of development.

It should be noted that there are two headings for each unit in the Contents. The first heading names the social situation which the authors have established for each unit of work. The second heading indicates the associated arithmetical concepts which are developed in the unit.

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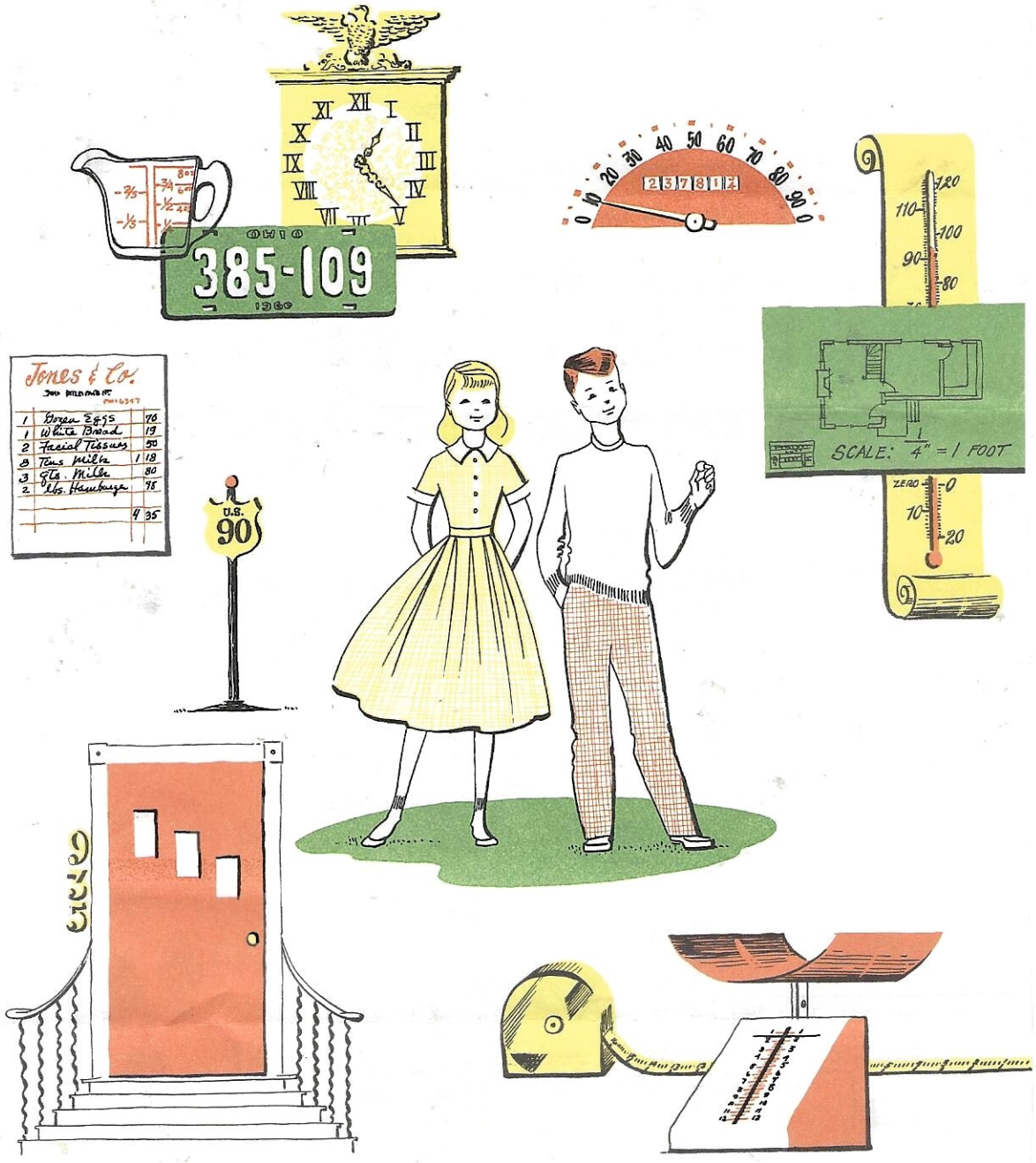
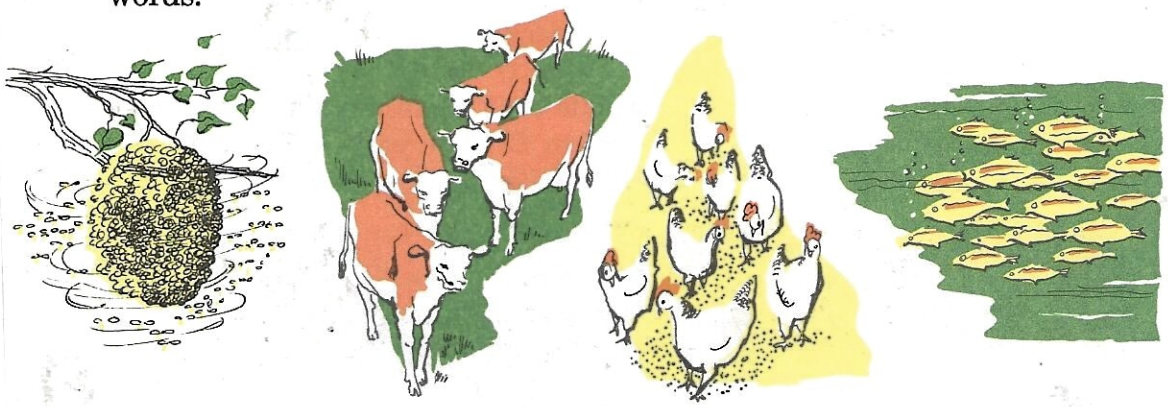
UNIT 1

How Numbers Work for Us



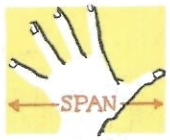
When man wants to describe a quantity exactly, he must use a number. But when he needs only a general description, without knowing the exact value, he can use words.

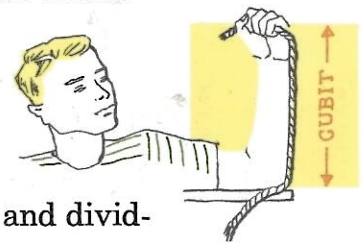


Did you ever stop to think of all the words that are used in place of numbers to describe a quantity of something? There are words like *some*, *many*, *few*, *a lot*, and then there are words like the following: *heap*, *drove*, *covey*. When are they used? Do you know the kinds of things the following *quantity words* are describing?

a flock of	?	a swarm of	?
a herd of	?	a school of	?

There are other words that man has used to describe a specific quantity. Some of them are found in the Bible.





1. About how long is the cubit in our measuring system?

2. Why did man need to learn to count?

3. Why are adding, subtracting, multiplying, and dividing called short-cuts for counting?

4. How can you express distances in a town or city? in the country? to the moon? to the North Star?

5. How many pounds in a short ton? in a long ton?

6. How many feet in a rod? in a mile?

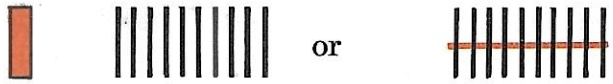
Checking Some Understandings

When you count things and say the word *thirty-two*, you are speaking a *number name* which tells the *how many* in the group. The *symbol* for this is 32, which is called a *number*.

Now, the number name tells that there are thirty and two things. Do you see the 30 in 32? Do you see the two? This is how the 3 may stand for the thirty in our thinking:

10 ones

Do three dimes equal thirty pennies? Why? Can we say the following?



1 ten 10 ones or a bundle of 10 ones

If we can, then:

30 ones

3 tens =

Then 32 = 3 tens (or 30 ones) + 2 ones.

In 32, the place held by the 3 is **tens place**, the place held by the 2 is **ones place.**

1 hundred 10 tens 100 ones

Can the place on the left of tens in any number be called by the name **hundreds place**? Why?

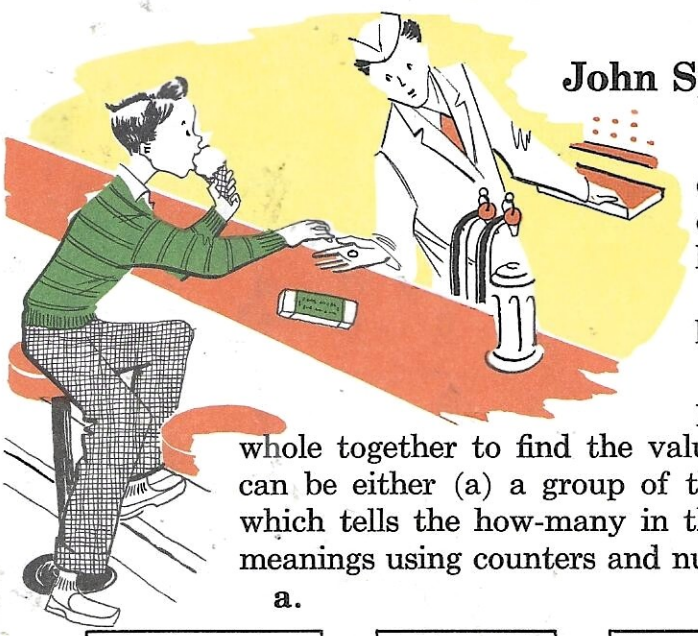
1. 24 = 2 tens and 4 ones = 20 ones and 4 ones. Why?

2. 375 = 36 tens and 15 ones = 3 hundreds and 75 ones. Why?

Do You Know These Facts?

Write the answers to these addition combinations on the strips of a folded sheet of paper. Check your work with another pupil. You should know the answers at sight.

2	2	0	6	0	8	6	0	4	3
4	2	2	2	4	3	0	8	5	0
6	1	8	9	1	4	3	6	5	9
4	4	2	0	5	3	2	6	0	1
0	3	5	1	2	8	0	7	1	0
9	6	5	7	1	1	7	2	9	1
8	9	1	1	1	2	7	6	3	5
8	2	1	3	6	0	7	1	3	4
2	4	2	2	7	1	9	4	4	3
8	4	9	5	1	8	9	0	1	1
3	6	3	2	3	3	6	9	2	0
8	5	4	3	9	5	3	3	6	6
1	5	4	7	5	8	2	4	1	8
2	3	2	3	2	4	7	8	0	0
0	6	9	6	0	5	7	5	7	8
3	9	7	8	5	8	0	1	9	5
5	7	4	7	4	8	5	9	8	6
7	8	9	5	7	6	9	6	9	7
8	4	7	5	0	7	9	9	3	9
7	6	4	6	0	6	8	5	7	4



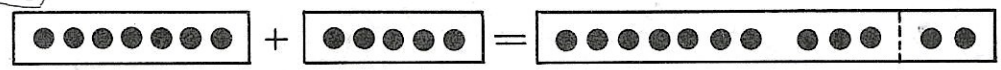
John Spends Some Money

John spent 7¢ for an ice cream cone and 5¢ for a candy bar. How much did he spend altogether?

Is this an addition problem?

Addition is putting the parts of some unknown whole together to find the value of that whole. A part can be either (a) a group of things, or (b) the number which tells the how-many in the group. Examine these meanings using counters and numbers:

a.



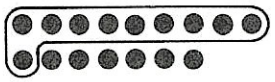
b. $7 + 5 = 7 + 3 + 2 = 10 + 2 = 12$

John spent 12¢ for ice cream and candy.

The sign + is used to indicate addition. The answer of an addition exercise is called the *sum*.

Add:

$$\begin{array}{r} 9 \\ +\,7 \\ \hline \end{array}$$



If you don't know the fact $9 + 7$, you can regroup the 7 to make $9 + 1 + 6 = 10 + 6 = 16$.

You see that $9 + 7 = 1$ ten and 6 ones. This is called **building a ten.**

1. Add, using the names of the places:

$$\begin{array}{rcr} 48 & = & 4 \text{ tens} + 8 \text{ ones} \\ +\,37 & & +\,3 \text{ tens} + 7 \text{ ones} \\ \hline 85 & & 7 \text{ tens} + 15 \text{ ones} \end{array}$$

or

8 tens + 5 ones = 85

Do you already know that 8 ones and 7 ones are 15 ones, or do you need to build a ten? $8 + 7 = 8 + 2 + 5 = 15$

How is 7 tens + 15 ones changed into 8 tens + 5 ones?

2. Add by building tens:

$$\begin{array}{r} 8 \\ 9 \\ +\,7 \\ \hline ? \end{array}$$

You think: 8 ones + 9 ones = ?
8 + 9 = 8 + 2 + 7 = 10 + an ending of 7 = 17
17 + 7 or 1 ten and 7 ones + 7 ones = ?
You must build another ten: 17 + 7 = 17 + 3 + 4 = 20 + an ending of 4 = 24.

In adding these numbers, the 17 + 7 is called a hidden combination. Do you know why?

Use the steps given above to add these:

7	6	8	9	5	6	7	4	9	9
5	9	8	5	8	7	7	9	3	9
9	7	8	7	8	8	7	9	9	9

Do you know what the ending of a number is? It is the figure in ones place of all numbers with two or more places. 5 is the ending of 25.

3. What are the endings in these additions?

15	25	35	15	9	19	29	39	28
2	2	2	12	6	6	6	16	19

You use bridging in addition when you add from one tens number to the next. For example, if you add from a teens number to a twenties number like this, 15 + 9 = 15 + 5 + 4 = 20 + 4 = 24, you have bridged 20.

This is called **bridging tens.**

If you build tens in adding columns of figures, you always find an ending and know which tens you are bridging.

4. Add, using bridging and endings:

$$\begin{array}{r} 28 \\ 69 \\ +\,57 \\ \hline 4 \end{array}$$

Adding the ones: 8 ones + 9 ones = ? ones.
8 + 9 (bridging 10 and ending in 7) = 17.
Why? 17 ones + 7 ones = ? ones. (You bridge 20. Why?) What is the ending?
Finish the exercise.

Practice in Addition

1. Write the hidden combinations that you find in adding these sums:

9	3	5	8	7	7	8	8	9
7	5	4	7	9	9	7	7	6
5	3	2	5	5	6	6	7	4
3	6	7	2	4	6	8	5	8

2. Add by writing the names of the places:

58	84	86	76	55	84	62	43	99
75	47	53	97	78	89	49	64	73

3. Add by building tens:

7	2	7	3	3	4	9	7	3
6	3	4	7	5	4	3	8	8
9	9	3	4	8	2	5	6	6
5	1	5	8	4	9	2	3	6

4. Add, using bridging and endings in both columns:

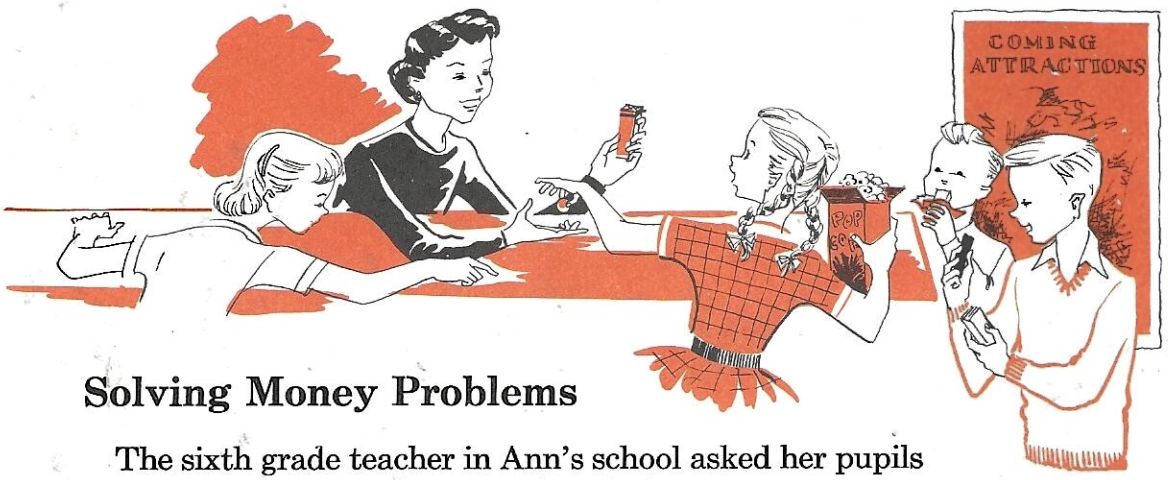
27	74	42	49	47	10	73	80	63
73	38	71	23	25	89	7	98	8
46	27	83	50	36	50	9	46	47
38	20	53	37	70	49	12	33	36

5. Add these as combinations by seeing the endings and bridging:

15	19	23	28	39	36	53	46	66
8	7	9	6	4	7	8	9	6

6. 3 ft. and 5 in. + 6 ft. and 8 in. = 9 ft. and 13 in. = ? ft. and 1 in.

8 ft. and 7 in. + 9 ft. and 9 in. = ? and ? = ?



Solving Money Problems

The sixth grade teacher in Ann's school asked her pupils to report on some of the ways that their money was spent last week.

1. Ann said that she spent 15¢ for a box of popcorn and 5¢ for a package of gum during a movie. How much money did she spend in all?

2. John said that he bought 3 prizes for his party. He paid 35¢ for the first prize, 25¢ for the second prize, and 10¢ for the booby prize. How much did he pay for all three prizes?

3. Jane bought a present for her grandfather's parakeet. She could not decide between a bell for 19¢ and a mirror for 29¢. How much did she have to spend to buy both?

4. Each child in the class was asked to buy a pencil, an eraser, a ruler, and a tablet. How much money did each person spend if the pencil cost 6¢, the eraser cost 10¢, the ruler cost 29¢, and the tablet cost 15¢?

5. Susan bought a pound of butter for 69¢ and a loaf of bread for 22¢ for her mother. How much money did she give the clerk?

7. Helen made a dress for her doll. The pattern cost $.35, the material cost $.69, and the thread and lace trim cost $.27. How much money did she spend altogether?

8. Betty bought a horn for her bike costing $1.25 and a light costing $3.49. How much did she spend for these two items?

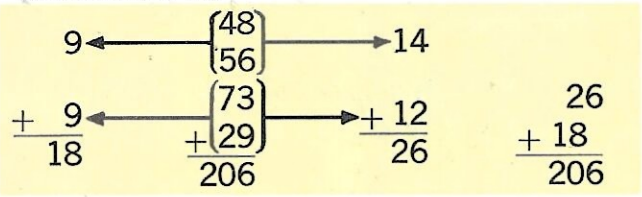
Checking Addition

Most people check an addition by adding in the opposite direction from the one used to find the sum. That is, if they add down to find the sum, they will add up to check.

1. Add and check:

28	30	73	68	9	28	33	19	26
77	27	63	44	58	7	48	38	60
66	76	89	76	69	40	57	44	88

2. Here is another way to check an addition. See if you understand it:



Why must the numbers be placed as they are?

3. Are estimates like the one given below a good method of checking, or are they just a way of deciding if an answer is reasonable?

49	Estimate:	50
36		40
+ 52		+ 50
137		140

Why is 50 rather than 40 used for 49? Why are the other two numbers used?

4. Can you check 49 + 36 by using estimated values for the numbers?

```
 49
+36
 85
```

Check: 50 + 40 = 90. You know that 90 is 5 too large, so you say 90 – 5 = 85. Why?

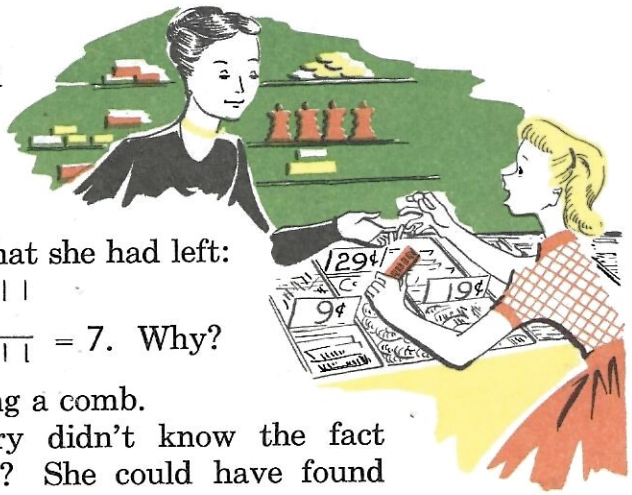
5. Add these, and use the method in Ex. 4 to check the sums:

```
 47
+32
```

Check: 50 + 30 – 1 = ? (Why must 1 be subtracted?)

66	49	68	55	27	88	7	60	33
28	29	28	37	64	9	38	45	37

Mary Uses Subtraction



Mary had 16¢ and spent 9¢ for a comb. How much money did she have left?

This is how Mary showed what she had left:

$16 - 9 =$ | | |

Do You Know These Subtraction Facts?

Write the answers to these subtraction combinations on the strips of a folded sheet of paper. Check your answers with another pupil. (You should know all of the subtraction facts in this list.)

7	8	9	6	7	9	12	11	8	10
3	8	3	0	2	0	6	2	6	7
9	6	9	4	8	4	5	1	9	5
1	6	8	4	1	2	4	1	9	3
9	5	8	10	3	3	0	3	5	7
6	0	2	4	0	3	0	2	1	7
6	5	3	4	2	3	7	6	8	5
3	2	1	3	1	1	5	5	4	5
6	2	7	8	4	7	9	6	10	11
4	2	5	7	1	6	5	2	2	8
9	10	14	10	11	8	13	11	15	10
2	3	7	9	7	5	6	5	8	1
10	12	7	8	10	12	14	10	7	12
5	9	0	3	6	4	6	8	4	8
1	14	16	13	14	8	17	9	2	9
0	9	9	4	5	0	9	7	0	4
4	16	13	11	13	15	13	13	17	15
0	7	5	3	7	9	9	8	8	7
11	12	15	14	12	11	12	11	16	18
9	3	6	8	5	4	7	6	8	9

Meanings in Subtraction

A. Take-Away Method

1. How to subtract, using the names of the places:

$$\begin{array}{r} 32 \\ -\ 18 \\ \hline 14 \end{array} = \begin{array}{r} \text{3 tens and 2 ones} \\ -\ \text{1 ten and 8 ones} \\ \hline \end{array} = \begin{array}{r} \text{2 tens and 12 ones} \\ -\ \text{1 ten and 8 ones} \\ \hline \text{1 ten and 4 ones} = 14 \end{array}$$

$$\begin{array}{r} 625 \\ -\ 248 \\ \hline 377 \end{array} = \begin{array}{r} \text{5 hundreds and 11 tens and 15 ones} \\ -\ \text{2 hundreds and 4 tens and 8 ones} \\ \hline \text{3 hundreds and 7 tens and 7 ones} = 377 \end{array}$$

2. How to subtract, thinking place values:

$$\begin{array}{r} {\scriptstyle 1\,15} \\ 6\not{2}\not{5} \\ -\ 248 \\ \hline 7 \end{array}$$

You cannot subtract 8 ones from 5 ones. You must have more ones. Use 1 of the 2 tens. 1 ten + 5 ones = 10 ones + 5 ones = 15 ones. 15 ones − 8 ones = 7 ones. Write 7 in ones place in the answer.

$$\begin{array}{r} {\scriptstyle 5\,11\,15} \\ \not{6}\not{2}\not{5} \\ -\ 248 \\ \hline 77 \end{array}$$

You cannot subtract 4 tens from 1 ten. You must have more tens. Use 1 of the 6 hundreds. 1 hundred + 1 ten = 10 tens + 1 ten = 11 tens. 11 tens − 4 tens = 7 tens. Write 7 in tens place in the answer.

$$\begin{array}{r} 625 \\ -\ 248 \\ \hline 377 \end{array}$$

5 hundreds − 2 hundreds = 3 hundreds. Write 3 in hundreds place in the answer.

The answer is 3 hundreds, 7 tens, and 7 ones, or 377.

B. Addition Method

3. How to subtract, finding how many more:

$$\begin{array}{r} 64 \\ -\ 26 \\ \hline \end{array}$$

First step:

$$\begin{array}{r} 14 \\ -\ 6 \\ \hline ? \end{array}$$

What do you add to 6 ones to give an ending of 4 ones in the sum? 8 + 6 = 14. When you are thinking this ending of 4 (in 14), you are using 1 of the 6 tens. Why?

What added to 2 makes 5? Why 5?

Practice in Subtraction

1. Why do you sometimes change a ten to ones when doing subtractions?

2. When is it necessary to use 1 hundred as 10 tens in subtraction?

3. Why can the answer in subtraction be called the **difference**?

4. Subtract, writing the names of the places (use the take-away method):

94	93	83	98	64	56	37	60	86
63	49	56	49	37	49	8	29	79

258	185	648	225	202	347	605	333	422
150	123	156	118	156	257	408	155	198

5. Subtract, thinking the place values:

88	95	72	44	50	70	84	66	75
55	21	35	29	32	40	79	66	50

200	86	72	118	223	278	385	405	500
100	53	49	58	116	186	198	268	275

6. Give the ending *only* in these subtractions:

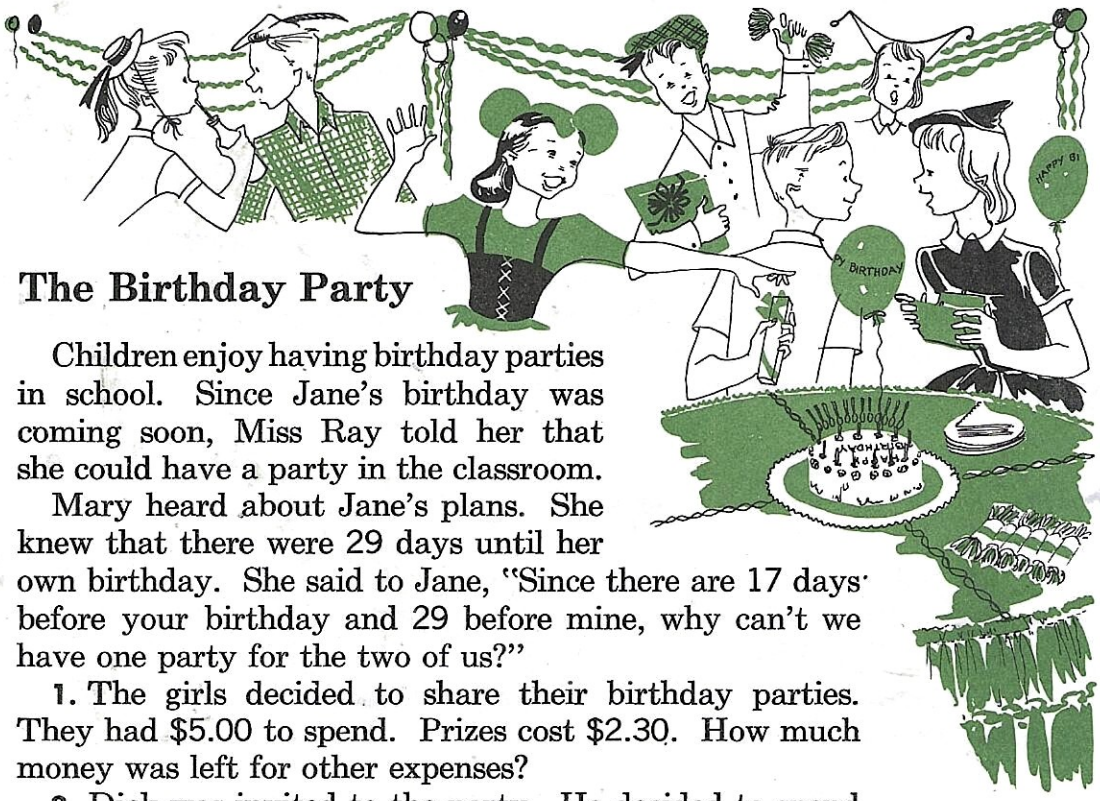
34	46	84	73	63	55	158	235	490
19	27	67	25	48	19	127	126	208

7. Use the addition method of subtraction in these:

54	66	42	47	64	82	70	69	50
32	34	27	39	28	29	56	49	30

8.

10 ft. and 9 in.	8 ft. and 6 in.	12 ft. and 4 in.
– 6 ft. and 4 in.	– 3 ft. and 6 in.	– 7 ft. and 9 in.



The Birthday Party

Children enjoy having birthday parties in school. Since Jane's birthday was coming soon, Miss Ray told her that she could have a party in the classroom.

Mary heard about Jane's plans. She knew that there were 29 days until her own birthday. She said to Jane, "Since there are 17 days before your birthday and 29 before mine, why can't we have one party for the two of us?"

1. The girls decided to share their birthday parties. They had \$5.00 to spend. Prizes cost \$2.30. How much money was left for other expenses?

2. Dick was invited to the party. He decided to spend \$1.25 for gifts. His bank account was \$4.27 before buying the gifts. How much was it after his purchases?

3. Johnny saw a coin purse that he knew Jane would like. It cost 89¢. He had \$1.40 to buy presents for both Mary and Jane. How much would he have left for Mary's present if he bought the coin purse for Jane?

4. Mary's mother gave the girls 10 yd. of crepe paper for decorations. They used 4 yd. to put trimming around a table. How many yards did they have left?

5. The girls invited all of their classmates. 18 of the 30 children came. How many were absent?

6. After the party Pete and Jack went to a movie. They had 88¢ left after they had bought their presents for the girls. If the tickets to the movie cost 50¢, did they have enough money left to pay 30¢ for 2 bus rides home?

Checking Subtraction

Does knowing the related number facts in addition and subtraction give you a method of checking the answer to a subtraction exercise? Why?

Knowing the addition method as well as the take-away method in subtraction will also help you. Do you see that one method uses the related fact in the other? So, if you use take-away, then check by using addition facts.

1. Subtract and check:

$$\begin{array}{r} 92 \\ -\,47 \\ \hline ?\ \text{and}\ ? \end{array} = \begin{array}{r} 8 \text{ tens and } 12 \text{ ones} \\ -\,4 \text{ tens and } 7 \text{ ones} \\ \hline \end{array} = ?$$

Check	Check
92	47
$-\ ?$	$+\ ?$
47	92

2. Subtract these, checking two ways:

88	59	68	72	167	158	216	328	400
26	39	40	55	97	67	97	159	208

3. How can estimating an answer in subtraction be used as a reasonable check?

Study this:

$$\begin{array}{r} 84 \\ -\,39 \\ \hline 45 \end{array} \qquad \text{Estimating:} \quad \begin{array}{r} 80 \\ -\,40 \\ \hline 40 \end{array}$$

Why did we use these numbers in estimating?

Would it be just as easy to estimate the answer by using $84 - 40$? Why?

4. How does the clerk at a store count your change if you give him more in payment than the article costs? How can you use the take-away method of subtraction to check the change that you receive?

Why is the clerk's method like the addition method in subtraction?

5. Can you check the subtraction $94 - 48$ by using $94 - 50 + 2 = 44 + 2 = 46$? Why? Can you check by using $90 - 50 + 6 = 40 + 6 = 46$? Why?

6. John said, "$697 - 304 = 203$." Can you check this without writing the exercise? Is his answer right?

Practice Exercises and Problems

1. Add:

69¢	38¢	56¢	55¢	50¢	25¢	83¢	7¢
29¢	24¢	27¢	36¢	37¢	45¢	9¢	78¢

2. Subtract:

58¢	67¢	54¢	96¢	74¢	85¢	90¢	63¢
24¢	27¢	36¢	28¢	28¢	36¢	11¢	9¢

3. Tell why a number like \$1.35 can be considered as having ones, tens, and hundreds places. Why is 1 the number of whole dollars? Why is the number on the right of the cents point a part of a dollar?

Add:				Subtract:			
\$.64	\$1.56	\$2.37	\$4.04	\$.89	\$.66	\$1.78	\$2.28
.27	.88	.96	1.27	.54	.59	.92	1.59

4. When using the sign (\$) to tell dollars and the point (.) to tell cents, you must always be careful to put them in their right places in any addition or subtraction. Can you think of a rule about where to place cents points?

5. Harry spent \$2.45 for a pair of gym shoes, \$.79 for a "beanie" cap, \$.87 for a belt, and \$.98 for a pocket knife. How much did he spend altogether?

6. Sam wants a leather jacket. If the one he wants costs \$9.49 and he now has only \$5.67, how much more money must he have to buy the jacket?

7. Dot needs a notebook that costs 75¢ and a ballpoint pen that costs \$1.69. If she has \$2.50, will this amount be enough to meet her needs? Will she have any money left? If so, how much?

8. 4 ft. and 9 in.
+ 3 ft. and 6 in.

9. 7 ft. and 8 in.
− 4 ft. and 10 in.

Joe Builds a Dog House

Joe decided to build a house for his young pup. He needed 6 boards each 14 in. long. What total length of board (in inches) will give him what he needs?

Is this a multiplication problem? Can you do it by addition? Does $6 \times 14 = 14 + 14 + 14 + 14 + 14 + 14$? Why?

Multiplication really comes from addition. If you can remember the answer when you add a group of the same sized numbers, this is multiplication. For example, when you find that $7 + 7 + 7 + 7 + 7 = 35$, then you know at once that $5 \times 7 = 35$.

When the size of the number in the addition group is large, we don't usually remember the sum. So our process of multiplication has to be something besides remembered addition. Let's examine Joe's problem:

Meanings

$$\begin{array}{r} 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ +\ 14 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 14 \\ \times\ 6 \\ \hline ? \end{array}$$

Multiplication by Parts

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline \end{array} = \begin{array}{r} 1 \text{ ten and } 4 \text{ ones} \\ \times\ 6 \\ \hline 6 \text{ tens and } 24 \text{ ones} \end{array}$$

or

$$8 \text{ tens and } 4 \text{ ones} = 84$$

Here's how to do the multiplication by just thinking the place values:

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline 4 \end{array}$$

6×4 ones $= 24$ ones. 24 ones $= 2$ tens and 4 ones. Write 4 in ones place in the answer, and hold the 2 tens to put with other tens which we are going to find (6×1 ten).

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline 84 \end{array}$$

6×1 ten $= 6$ tens. 6 tens $+ 2$ tens (held) $=$ 8 tens. Write 8 in tens place in the answer.

Joe needs a board 84 in. long.

Multiplying by a Two-Place Number

After Joe started building his dog house, he saw that he would need twice as many boards as he had first planned. Thus, he needed 12 of them. How can he find the total length of board needed for twelve 14 in. pieces?

Can he double the answer that he found for 6 pieces? Can he add $84 + 84$? Why? Let's do the work by multiplication.

$$\begin{array}{r} 14 \\ \times\ 12 \\ \hline ? \end{array}$$

14 = 1 ten and 4 ones; he could take each of these parts 12 times separately and then put them together.

Do you know twelve 4's?

Will it be easier to use 12 as 10 and 2?

$$\begin{array}{r} 14 \\ \times\ 12 \\ \hline ? \end{array} = \begin{array}{r} 1 \text{ ten} + 4 \text{ ones} \\ \times\ 2 \\ \hline 2 \text{ tens} + 8 \text{ ones} \end{array} + \begin{array}{r} 1 \text{ ten} + 4 \text{ ones} \\ \times\ 10 \\ \hline 10 \text{ tens} + 40 \text{ ones} \end{array}$$

or

14 tens + 0 ones Why?

or

1 hundred + 4 tens + 0 ones Why?

When you add these parts together, you have:

$$\begin{array}{r} 2 \text{ tens} + 8 \text{ ones} \\ +\ 1 \text{ hundred} + 4 \text{ tens} \\ \hline 1 \text{ hundred} + 6 \text{ tens} + 8 \text{ ones} = 168. \end{array}$$ Why?

Joe would need 168 in. of board for the 12 pieces.

The answer to a multiplication exercise is called the **product**. The parts that are added to give the product (the final answer) are called the **partial products**.

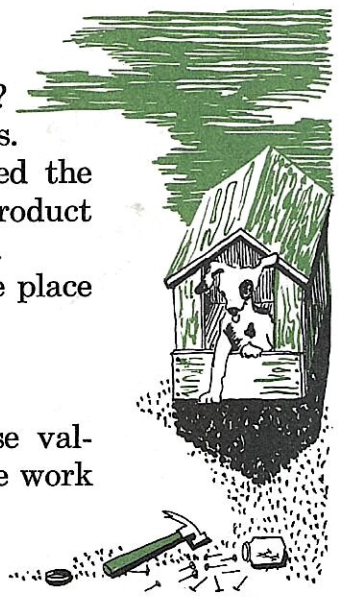
Can you multiply 14 by 12 just by thinking the place values?

$$\begin{array}{r} 14 \\ \times\ 12 \\ \hline 28 \\ 140 \\ \hline 168 \end{array}$$

28 first partial product
140 second partial product
168 product

$2 \times 14 = ?$
$10 \times 14 = ?$

Explain how these values are placed in the work at the left.



Multiplication—The General Method

See if you can use the meanings found in thinking place values on the previous page to discover a shorter way of doing multiplications.

$$\begin{array}{r} 46 \\ \times\ 12 \\ \hline ? \end{array}$$

Remember: 12×46 means to take the number 46 twelve times, either by addition or multiplication. Not knowing the 12's facts in multiplication, you can multiply by 12 only by using parts of the number which you know. Can you use 10 and 2?

$$\begin{array}{r} 46 \\ \times\ 12 \\ \hline 2 \end{array}$$

2×6 ones = 12 ones = 1 ten + 2 ones.

You put 2 in ones place in the first partial product and hold the 1 ten until you find 2×4 tens. Then you add them.

$$\begin{array}{r} 46 \\ \times\ 12 \\ \hline 92 \end{array}$$

2×4 tens = 8 tens. 8 tens + 1 ten (held) = 9 tens. Write 9 in tens place in this first partial product.

Without thinking values:

You can say, $2 \times 6 = 12$ or two 6's = 12. The 2 in the 12 holds the same place in the first partial product as the 6 holds in 46. This is true because it is that place that is being multiplied. The 1 in the 12 is held until you have $2 \times 4 = 8$. Then 8 + 1 (held) = 9. The 9 is put in this partial product in the same place as 4 in 46. Why?

Multiplying by 10:

If you think values:

$$\begin{array}{r} 46 \\ \times\ 12 \\ \hline 92 \\ 6 \end{array}$$

10×6 ones = 60 ones. You can write 0 in ones place and 6 in tens place in the second partial product if you wish. But, do you really need to write the 0? Why or why not?

$$\begin{array}{r} 46 \\ \times\ 12 \\ \hline 92 \\ 46 \\ \hline 552 \end{array}$$

10×4 tens $= 40$ tens $= 4$ hundreds. Write 4 in hundreds place in this partial product. Do you see that you could have held the 6 tens and added them to the 40 tens to get 46 tens, or 4 hundreds + 6 tens?

Without thinking values:

$1 \times 6 = 6$. This 6 holds a higher place in the partial product than the 6 in 46. Can you tell why? (Remember, this 1 is 1 ten, or 10.)

$1 \times 4 = 4$. Why is it put in the partial product in a place higher than the 4 in 46?

See if you can answer the questions in this example:

Multiply, using the general method:

$$\begin{array}{r} 36 \\ \times\ 24 \\ \hline 144 \\ 72 \\ \hline 864 \end{array}$$

$4 \times 6 = 24$. How is this value recorded? $4 \times 3 = 12$. How do you get 1 in hundreds place and 4 in tens place out of the work?

$2 \times 6 = 12$. Why is the 2 put in tens place? What happens to the 1? $2 \times 3 = 6$, $6 + 1 = 7$ for hundreds place. Why?

1. $\begin{array}{r} 36 \\ \times\ 2 \end{array} = \begin{array}{r} 3 \text{ tens} + 6 \text{ ones} \\ \times\ 2 \end{array} = ?$

2. $\begin{array}{r} 128 \\ \times\ 3 \end{array} = \begin{array}{r} 1 \text{ hundred} + 2 \text{ tens} + 8 \text{ ones} \\ \times\ 3 \end{array} = ?$

3. $\begin{array}{r} 2 \text{ ft.} + 6 \text{ in.} \\ \times\ 3 \end{array} = \begin{array}{r} 2 \text{ ft.} + 6 \text{ in.} \\ \times\ 3 \end{array} = 7 \text{ ft.} + 6 \text{ in.}$ Why?

$6 \text{ ft.} + 18 \text{ in.}$

4. Multiply:

47	24	56	48	164	32	152	18	25
5	12	13	8	4	25	6	18	25

Do You Know These Facts?

Write the answers to these multiplication combinations on the strips of a folded sheet of paper. Check your answers with another pupil. Learn any that you do not remember.

0	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9
2	2	2	2	2	2	2	2	2	2
0	1	2	3	4	5	6	7	8	9
3	3	3	3	3	3	3	3	3	3
0	1	2	3	4	5	6	7	8	9
4	4	4	4	4	4	4	4	4	4
0	1	2	3	4	5	6	7	8	9
5	5	5	5	5	5	5	5	5	5
0	1	2	3	4	5	6	7	8	9
6	6	6	6	6	6	6	6	6	6
0	1	2	3	4	5	6	7	8	9
7	7	7	7	7	7	7	7	7	7
0	1	2	3	4	5	6	7	8	9
8	8	8	8	8	8	8	8	8	8
0	1	2	3	4	5	6	7	8	9
9	9	9	9	9	9	9	9	9	9

Boy Scout Activities

1. Jimmy's scout troop decided to raise money so that they could go to the state jamboree. The troop's first plan was to collect and sell old newspapers. They could be sold at 12 dollars a ton.

If the scouts can collect 24 tons in the 6 months before the date of the jamboree, how much money will they receive from the sale of the paper?

2. Phil's patrol had a soft drink stand at the first football game in the fall. What was the troop's profit on 21 cases of drinks if the profit on one case was 40¢?

3. While on a trip to Chicago, Jimmy found an exceptionally good sale on packets of stamps. He bought some for himself and other boys in his scout troop. The packets were valued at $3.00, but were priced at only 89¢. How much would it cost him to buy a packet for himself and for 6 other boys in his troop?

4. Phil's father promised to take his patrol on an overnight camping trip if the boys would earn the money for the gas. How much would they have to earn if gas cost 32¢ per gal. and 11 gal. of gas were needed?

5. Joe's troop planned a picnic. The boys decided that each one was to pay an equal share of the expenses. Joe was asked to collect all of the money. If 22 boys were able to go and each one's share was 22¢, how much money should he have after making the collections?

Checking Multiplication

Multiplication is sometimes called a short process of addition. You should know that you can always use addition as a check in multiplication.

Sometimes, using addition as a check of multiplication may be long. Look at Ex. 1 below, and tell why checking by addition may sometimes be long or tiresome.

1. Multiply, checking by addition:

48	48	175	175	78	Check:
× 3	48	× 4	175	× 5	?
?	+ 48	?	175		
	?		+ 175		
			?		

Harry says he sees a way to shorten the work of long additions when checking a multiplication exercise. This is his work; examine it, and tell why it is correct:

48	12 = 4 + 4 + 4	48	192	or	192
× 12		× 4	192		× 3
96		192	+ 192		576
48			576		
576					

2. Use Harry's method to check the products you find for the following:

36	64	156	24	68	12	44	32
× 15	× 18	× 9	× 24	× 8	× 30	× 21	× 27

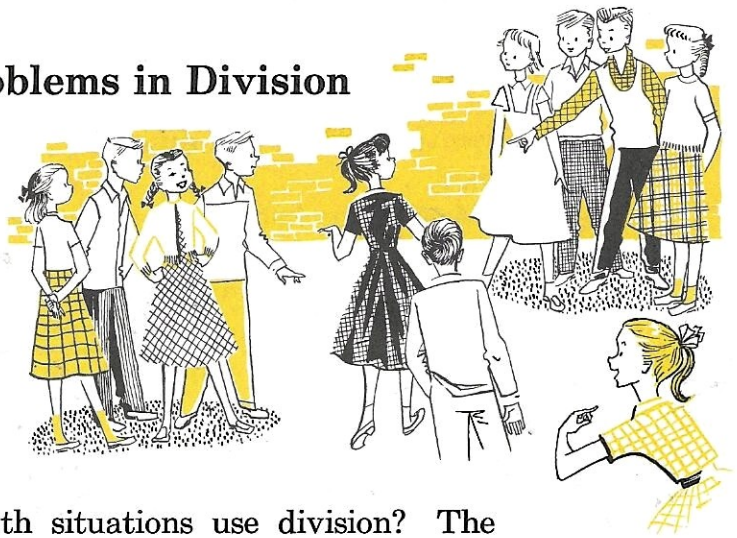
3. Can you use the following method for checking a product?

36	Check:			Explain the check.
× 24	36	36	36	
	× 10 +	× 10 +	× 4	

4. Multiply and check:

15	63	28	20	236	56	32	16
5	4	24	15	4	12	30	18

Two Kinds of Problems in Division



Miss Jones has 32 pupils in her class. There are times in the play period when the class is divided into 4 equal parts for games, and at other times it is divided into groups with 4 children in each group.

Do you see that both situations use division? The stick figures below are arranged to show the difference:

a. 32 pupils—4 equal parts

$4\overline{)32}$ 8

b. 32 pupils—groups of 4 pupils each

$4\overline{)32}$ 8

What is the difference between the answers for these two problems? *Always label your answers in division.* The answer for **a** is 8 *children*, while for **b** it is 8 *groups*.

1. How many groups of 6 children each are there in a group of 48 children?

2. How will you label your answer if you divide 48 children into 6 equal groups?

3. Divide 24 children into 3 equal groups. What is the size of one of the equal groups?

4. How many groups of 3 children each can you make from a group of 24 children?

5. Tell which of the above 4 problems were examples of finding the size of one of the equal parts in a whole and which were finding how many groups in a whole.

Seeing One Method in Division

Here is how the two kinds of division problems are done by one method.

Suppose Miss Jones had 35 pupils in her class. The two problems of dividing them might be:

a. 7 equal parts; how many pupils in each part?

b. Groups of 7 pupils; how many groups would there be? Let's look at **b** first: 7 pupils$\overline{)35 \text{ pupils}}$ = ? groups

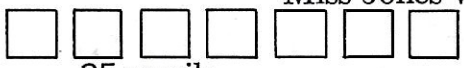
35 pupils	14 pupils	By subtraction, she could separate groups of 7 pupils, 5 times. So, there would be 5 groups of 7 in 35.
− 7 pupils	− 7 pupils	
28 pupils	7 pupils	
− 7 pupils	− 7 pupils	
21 pupils	0 pupils	
− 7 pupils		
14 pupils		

$$\begin{array}{r} 5 \\ 7\overline{)35} \\ -\,35 \\ \hline 0 \end{array}$$

When dividing, you see that the 35 can be divided evenly by 7.

This division is: *How many 7's in 35?* Now, examine **a**: 7$\overline{)35 \text{ pupils}}$ = ? pupils

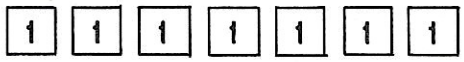
Miss Jones wants 7 equal parts.



35 pupils
− 7 pupils
28 pupils

She first makes 7 boxes, each one to represent one of the equal parts.

If she subtracts 7 pupils from the whole group, she will have 1 pupil to put into each box or part.



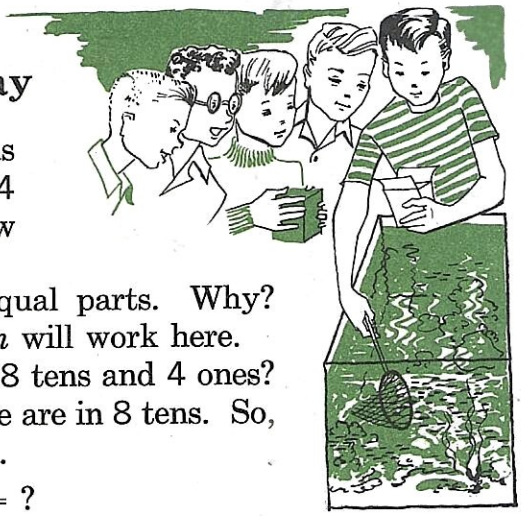
28 pupils
− 7 pupils
21 pupils

If she uses another group of 7, she will have 1 more pupil to put into each box or part.

She has now taken 2 groups of 7 out of 35, and she has 2 for each of the 7 equal parts.

She can repeat this process until all of the 35 are used. There will be as many in each part as the number of times 7 is subtracted. How many times? Notice that you now have the same question as in **b**: How many 7's are there in 35?

Dick Gives His Guppies Away



Dick is moving out of town and has decided to give his 84 guppies to his 4 friends. If he divides them equally, how many will each friend receive?

Dick wants to divide 84 into 4 equal parts. Why? Let's see if the *One Method in Division* will work here.

How many 4's are there in 84 or in 8 tens and 4 ones? It is not easy to see how many 4's there are in 8 tens. So, you can think of 84 as being 80 and 4.

$$4\overline{)84} = 4\overline{)80 + 4} = ?$$

How many 4's are there in 80? in 4?

$4\overline{)80} = ?$

If you know that there are ten 4's in 40, then you know that there are twice as many or 20 in 80. Why? Then your answer is $20 + 1 = 21$. Why?

$$\begin{array}{r} 20 + 1 \\ 4\overline{)80 + 4} \end{array}$$

Suppose you write the work in the usual way of doing a division. You can then think of the number of 4's in 8 tens instead of the number of them in 80.

$$\begin{array}{r} 2 \\ 4\overline{)84} \\ 8 \\ \hline 4 \end{array}$$

How many 4's are there in 8 tens? You already know that there are 20 or 2 tens of them in 80. So, there must be 2 tens of them in 8 tens. You can shorten your thinking even more this way: How many 4's are there in 8? Since 8 is in tens place, 2 must go in tens place in the answer.

When you write 8 under the 8 in 84, this shows that you have used the 8 tens or 80 out of the 84. There are 4 still to be used.

How many 4's in 4? Does the work show this? How?

$$\begin{array}{r} 21 \\ 4\overline{)84} \\ 8 \\ \hline 4 \\ 4 \\ \hline 0 \end{array}$$

The answer is 21 guppies.

This method, which can be used to solve all divisions, is called the **general method**. You must always know how to label your answer.

Estimating Quotients

A good way to find the first figure in the answer to a division exercise is by estimation. Divide $4\overline{)72}$ this way:

```
  10
4)72
  40
```

Are there as many as ten 4's in 72? Can there be 20 of them? Why? Will the quotient be a one- or a two-place number?

```
   1
4)72
   4
   3
```

If the quotient is a two-place number, then there will be a figure placed above the 7. Why?

```
  18
4)72
   4
  32
  32
```

If there are 10 (and not 20) of the 4's in the number, then you use 4 of the 7 tens and place a 1 over the 7 in the quotient. Why?

3 of the 7 tens have not been used. These, with the 2 ones in 72, give 32 yet to be divided. Why? How many 4's in 32?

Try another one. Divide $3\overline{)198}$:

```
3)198
```

198 is a three-place number. You can see that the quotient will not have 3 places. Why?

```
  60
3)198
 180
  18
```

Are there more than ten 3's in 198? more than 50? Perhaps the number of 3's in 19 will help you.

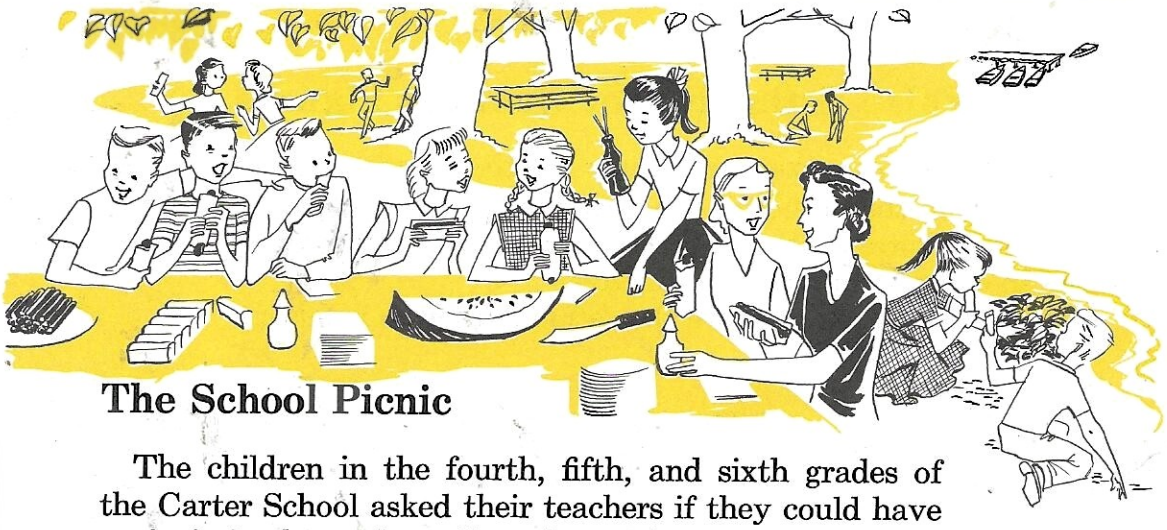
You know that $3\overline{)19} = 6$ with 1 left over. So there are at least sixty 3's in 198. 6 is placed over the 9 in the quotient. Why?

```
  66
3)198
 18
  18
  18
```

Would $3\overline{)19}$ be just as good a clue to the first quotient figure as estimating the number of 3's in 198?

Divide:

$3\overline{)69}$ $4\overline{)88}$ $2\overline{)64}$ $4\overline{)48}$ $2\overline{)86}$ $2\overline{)144}$ $2\overline{)176}$



The School Picnic

The children in the fourth, fifth, and sixth grades of the Carter School asked their teachers if they could have a picnic in the park on Saturday. They wanted to play games and have good things to eat.

Joe and some other children were to buy the rolls for the sandwiches. They needed 168. Since rolls can be bought by the dozen, Joe wanted to find how many dozen he should buy.

$12\overline{)168}$

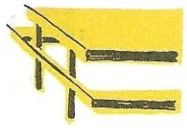
This is a division problem: $12\overline{)168}$. Why?

There are at least 10 twelves in 168. Why? $10 \times 12 = ?$

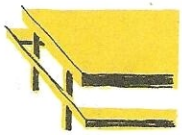
$$\begin{array}{r} 14 \\ 12\overline{)168} \\ \underline{12} \\ 48 \\ \underline{48} \end{array}$$

Why is the quotient a two-place number? Why is 12 out of the 16 used with 4 left over? Why is the 1 in the quotient placed over the 6 in 168? Explain the rest of the work.

Joe would need 14 dozen rolls.



Helen was asked to figure the number of tables that would be needed to seat all the children for lunch. She knew that each long table in the park would hold 32 children. If there were 96 children in all, how many tables would be needed?



$32\overline{)96}$

How many 32's in 96? Would $9 \div 3$ be a clue? Why? Will the quotient be a one- or a two-place number? Why?

Finish the division.

False Estimates for Quotient Figures

In some division exercises, if you use the figures as you find them in the numbers, the estimated quotient figures will not be correct. Sometimes you have to try different ones in order to find the true figures.

Here is an example where the estimated figure is false: Divide 468 by 18:

$18\overline{)468}$

The quotient is a two-place number. Why?

$$\begin{array}{r} 4 \\ 18\overline{)468} \\ 72 \end{array}$$

Looking at the 4 in 468 and the 1 in 18 for a clue, $4 \div 1 = 4$.

$$\begin{array}{r} 2 \\ 18\overline{)468} \\ 36 \\ \hline 108 \end{array}$$

Should you try 40×18 (putting 4 in tens place in the quotient), or should you begin with 10, then try 20, and so on?

18 as a divisor is almost 20. Perhaps a trial with 20 as a divisor might help you.

$$\begin{array}{r} 25 \\ 18\overline{)468} \\ 36 \\ \hline 108 \\ 90 \\ \hline 18 \end{array}$$

How many 20's in 468? Your two-place quotient should be estimated as a 20-number, giving 2 in tens place as the first quotient figure. Try it.

You now need to find the number of 18's in 108. By a trial process, $108 \div 20 = 5$. A quotient figure of 5 is false ($5 \times 18 = 90$); you would have a remainder that is too large. Why?

$$\begin{array}{r} 26 \\ 18\overline{)468} \\ 36 \\ \hline 108 \\ 108 \\ \hline \end{array}$$

Try 6: $6 \times 18 = 108$. Your quotient is 26. Why?

Divide:

1. $16\overline{)336}$ $28\overline{)700}$ $26\overline{)832}$ $15\overline{)675}$ $19\overline{)798}$

In $28\overline{)924}$, when you estimate the tens figure in the quotient by using 30 as the divisor, it is called **rounding** the number to the nearest 10.

Do You Know These Division Facts?

Write your answer on the strips of a folded sheet of paper, then check these answers with another pupil. Learn any that you do not know.

$1\overline{)5}$	$4\overline{)8}$	$7\overline{)14}$	$1\overline{)4}$	$5\overline{)25}$	$3\overline{)15}$	$7\overline{)35}$	$5\overline{)30}$	$2\overline{)18}$
$7\overline{)28}$	$8\overline{)0}$	$4\overline{)12}$	$3\overline{)3}$	$9\overline{)81}$	$7\overline{)42}$	$8\overline{)8}$	$1\overline{)3}$	$5\overline{)35}$
$4\overline{)16}$	$9\overline{)9}$	$7\overline{)21}$	$3\overline{)6}$	$7\overline{)49}$	$1\overline{)2}$	$8\overline{)16}$	$5\overline{)40}$	$2\overline{)16}$
$8\overline{)24}$	$9\overline{)0}$	$5\overline{)20}$	$3\overline{)9}$	$5\overline{)45}$	$9\overline{)72}$	$6\overline{)0}$	$2\overline{)14}$	$7\overline{)56}$
$9\overline{)63}$	$6\overline{)6}$	$8\overline{)32}$	$1\overline{)1}$	$5\overline{)15}$	$4\overline{)20}$	$7\overline{)7}$	$3\overline{)12}$	$7\overline{)63}$
$6\overline{)12}$	$3\overline{)0}$	$8\overline{)40}$	$7\overline{)0}$	$4\overline{)24}$	$5\overline{)10}$	$9\overline{)54}$	$8\overline{)56}$	$2\overline{)12}$
$8\overline{)64}$	$2\overline{)0}$	$3\overline{)24}$	$4\overline{)0}$	$4\overline{)36}$	$6\overline{)42}$	$9\overline{)27}$	$6\overline{)30}$	$2\overline{)8}$
$2\overline{)4}$	$1\overline{)9}$	$3\overline{)27}$	$1\overline{)7}$	$6\overline{)36}$	$3\overline{)18}$	$8\overline{)72}$	$9\overline{)18}$	$1\overline{)8}$
$4\overline{)4}$	$2\overline{)6}$	$8\overline{)48}$	$5\overline{)0}$	$2\overline{)10}$	$5\overline{)5}$	$4\overline{)28}$	$6\overline{)54}$	$9\overline{)45}$
$3\overline{)21}$	$2\overline{)2}$	$1\overline{)0}$	$1\overline{)6}$	$6\overline{)18}$	$4\overline{)32}$	$6\overline{)48}$	$9\overline{)36}$	$6\overline{)24}$

The answer to a division exercise is called the **quotient**. The number by which another number is divided is called the **divisor**. The number being divided is the **dividend**.

Practice in Division

1. Divide: $31\overline{)316}$. An estimate of 10 is accurate. Why? How many are left over?

2. Divide: $24\overline{)288}$

$$\begin{array}{r} 1 \\ 24\overline{)288} \\ \underline{24} \end{array}$$

A close estimate is that there are about ten 24's in 288 ($10 \times 24 = 240$). Write 1 in tens place in the answer. Why? Write 24 under 28 tens to show that they have been used. Finish the work.

3. What is the first quotient figure in each of these divisions?

$2\overline{)59}$ $3\overline{)87}$ $5\overline{)86}$ $4\overline{)98}$ $6\overline{)99}$ $5\overline{)60}$ $3\overline{)57}$ $5\overline{)95}$

4. Think place values in finding the quotients for these:

$3\overline{)66}$ $2\overline{)98}$ $7\overline{)98}$ $6\overline{)90}$ $5\overline{)80}$ $5\overline{)65}$ $6\overline{)72}$ $4\overline{)92}$

$2\overline{)146}$ $3\overline{)159}$ $5\overline{)155}$ $6\overline{)240}$ $7\overline{)294}$ $4\overline{)192}$ $4\overline{)376}$

$12\overline{)120}$ $30\overline{)300}$ $21\overline{)840}$ $23\overline{)713}$ $12\overline{)299}$ $24\overline{)528}$

5. Use the estimating process to find these quotients:

a. $24\overline{)552}$ Will the quotient be 20 and some more? Why?

b.
$$\begin{array}{r} 1 \\ 16\overline{)192} \\ \underline{16} \end{array}$$
Finish the work and explain it.

c. $12\overline{)288}$ Will 2 in tens place in the quotient be the estimated first quotient figure? Why?

d.
$$\begin{array}{r} 9 \\ 6\overline{)57} \\ \underline{54} \\ 3 \end{array}$$
Try 9 as a quotient. Why? This is an *uneven* division. You should write the answer as 9 **R** 3. The **R** stands for the **remainder**.

Problems

1. Mike can earn 75¢ an hour by mowing lawns. Mr. Robinson, the grocer, offered him $6.40 to work in his store on Saturdays. If Mike worked 8 hours in the store each Saturday, would he earn more money or less money by the hour than he would by mowing lawns?

2. One of Mike's jobs at the grocery was to put oranges from boxes into bags. There were to be 12 oranges in each bag. If each of the boxes holds 156 oranges, how many bags would he need for each box of oranges?

3. Mr. Robinson's price for cranberries was 30¢ a quart. If Mike received $4.20 for the berries that he sold one Saturday, how many quarts did he sell?

4. The girls' club needed money. They were given permission by Mr. Robinson to have a bake sale in his store. If they had 20 doz. cookies and sold the entire lot for $8.00, what was their price by the dozen?

5. How many 12's are there in 204? in 432?

6. Mr. Robinson told Mike that his sale on onions last week brought in $29.24. If there were 34 bags sold, all at the same price, what was the amount received per bag?

7. Mike keeps a record of the miles that he rides while delivering orders for Mr. Robinson. For the month of September his speedometer showed that he had ridden a total of 13 miles. At this rate, how many months would he have to work for Mr. Robinson to ride a total of 200 miles?

Checking Division

Mike said, "When I want to check $9\overline{)63}$ (7), I remember the multiplication fact $7 \times 9 = 63$." Do you think this is a good way to check the quotient, when Mike thinks that he is dividing 63 things into 9 equal parts? He's multiplying 7 things $\times$ 9 parts.

Suppose Mike had the division: How many groups of 9 things are there in 63 things? There are seven 9's in 63. Then his way, 7×9, does tell the number of groups.

When you know that $7 \times 9 = 9 \times 7 = 63$, can you use one fact to check the other? Can you subtract to check?

You must always know how to label the answer in a division.

1. Divide and check:

a. $9\overline{)72}$ (8) How many 9's are there in 72? Check: $9 \times 8 = ?$

b. $12\overline{)168} = ?$ Check: $12 \times ? = ?$

c. $8\overline{)848} = ?$ $8 \times ? = ?$

d. Could you do exercise **c** by parts as shown below and use the work as a check on the first division?

$$8\overline{)848} = 8\overline{)800} + 8\overline{)48} = ?$$

2. Mike's father traveled 768 miles by car in 3 days. If each day's distance was the same, how many miles did he travel per day?

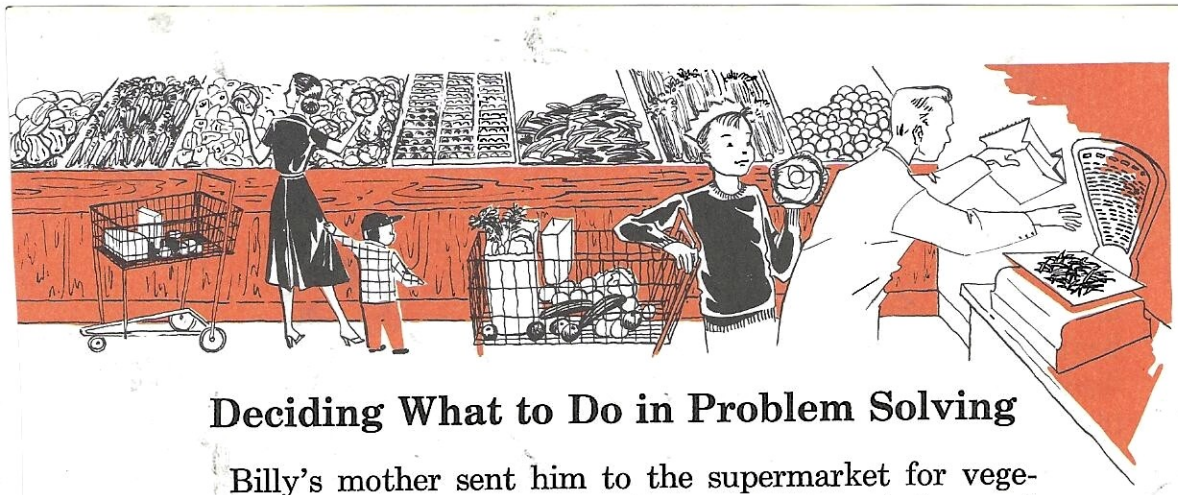
$3\overline{)768}$ (256)

You can see by an estimate that a quotient less than 200 and more than 300 would not be correct. Why?

3. Can an estimated quotient be a rough check on the correctness of a division example? How?

4. Divide and check:

$8\overline{)56}$ $4\overline{)76}$ $6\overline{)84}$ $7\overline{)84}$ $5\overline{)95}$ $4\overline{)104}$ $3\overline{)87}$



Deciding What to Do in Problem Solving

Billy's mother sent him to the supermarket for vegetables and fruit. Study each problem given below and tell:

a. what Billy needs to find.

b. what facts are given, whether more information is needed, and how it can be found.

c. what process Billy should use.

d. what answer, from a rough estimate, is sensible.

e. what check on the answer Billy must make.

1. Billy was told to order 60 lb. or 1 bu. of potatoes. Potatoes are selling at the rate of 100 lb. for $4.20. How much will the bushel of potatoes cost?

2. If Billy plans to buy 4 lb. of cabbage at 15¢ a pound and 2 doz. oranges at 45¢ a dozen, what will be their total cost?

3. Billy paid $1.87 for one bag of vegetables. If the bag contained tomatoes for 75¢, carrots for 39¢, cucumbers for 45¢, and lettuce for 28¢, was the amount that Billy paid correct?

4. What would be the best way for Billy to buy oranges: by the peck, bushel, or dozen?

5. How much change should Billy receive from a dollar bill in payment for a 69¢ bag of apples?

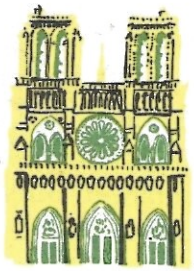
6. Billy bought 2 lb. of green snap beans and a quart of shelled lima beans. If the price for the snap beans was 29¢ per pound and the limas sold at 59¢ a quart, which item cost him the greater amount? How much greater?

Another Kind of Number

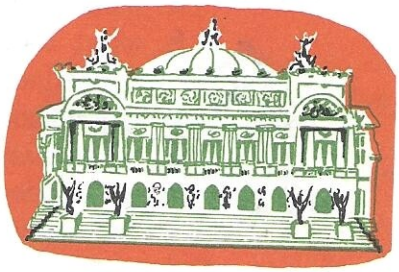
The symbols I, II, III, IV, V, VI, VII, VIII, IX, X are **Roman numerals** for the numbers 1 through 10. You may have used them already when you were making or reading outlines for some of your studies. Perhaps you have seen them when you were reading the time of day from the face of an old clock. You may have seen them in books or on statues.

Have you ever noticed something like MDCCCXLVI on the cornerstone of a building?

The Pantheon, Rome
XXVII B.C.



Notre Dame, Paris
MCLXIII



The Opera, Paris
MDCCCLXXIV

Find the age of the above buildings.

You can change Roman numerals to Arabic numbers. Study these and see what you can do with them:

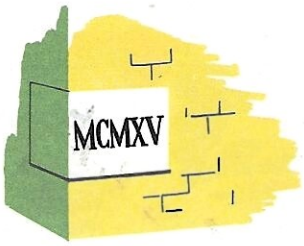
Roman	I	II	III	IV	V	VI	VII	VIII	IX	X
Arabic	1	2	3	4	5	6	7	8	9	10

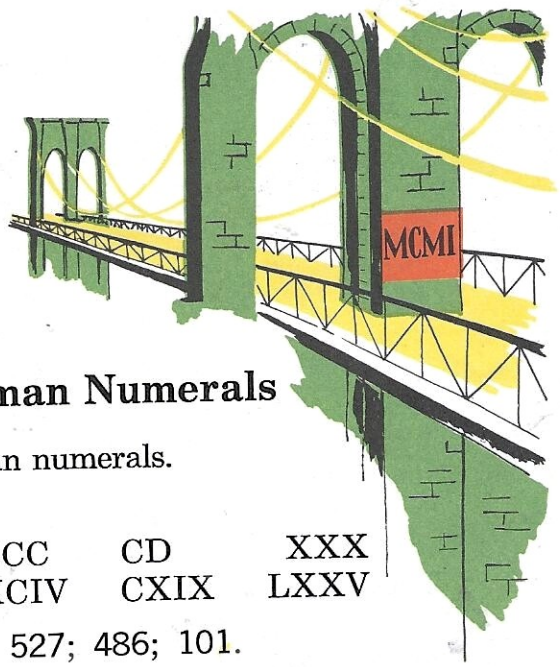
Roman	XI	XIV	XV	XVI	XIX	XX	XL	L	C	D	M
Arabic	11	14	15	16	19	20	40	50	100	500	1000

Do you understand how the teens numbers are written?

Compare the forms for 40 and 50. Why do you think X is placed before the L?

Why is XC the Roman form for the Arabic 90?





Exercises Involving Roman Numerals

1. Write your age using Roman numerals.

2. Write in Arabic numbers:

XVIII	XXIX	XLIX	CCC	CD	XXX
DCXL	MDC	DCCL	XCIV	CXIX	LXXV

3. Write in Roman numerals: 527; 486; 101.

4. Change the following to Arabic numerals and multiply:

XXII × XLVIII XIV × LV

5. Change the following to Arabic numerals and add:

CXV	CIV	CCXC	XIII	DXVI	CCXIV
CXL	CL	CCCX	IX	XIX	XVII
D	XXIX	CD	IV	CCC	XXXI

6. Change the following to Arabic numerals and subtract:

CX	CCL	XXXVIII	D	XL
XC	CLC	XIX	CCCLXVIII	XV

7. Change the following to Arabic numerals and divide:

XVI)XLVIII X)CC IV)XII C)DC D)M XL)CD

8. Do you think it was easier for the Romans to add, subtract, multiply, or divide with their numbers than it is for us with our numbers? Why or why not?

Unit Review

1. Sam sells papers in his home town. He sold 156 papers one week and 231 papers another week. How many more papers were sold during the second than the first week?

2. In Sam's town there are 283 homes without TV sets and 783 homes which have them. Which group is larger? How much larger?

3. Sam wants to earn $98.00 to buy a new boat. He earned $17.00 one week, $12.00 another week, and $27.00 a third week. How much more money must he earn?

4. Add:

67	427	604	802	6 ft. and 3 in.
43	265	243	59	5 ft. and 10 in.

5. Subtract:

750	258	654	680	12 ft. and 7 in.
607	136	327	494	8 ft. and 11 in.

6. Multiply:

24	48	148	236	2 ft. and 8 in.
12	15	2	4	4

7. Divide:

$4\overline{)92}$ $3\overline{)879}$ $8\overline{)656}$ $12\overline{)492}$ $36\overline{)868}$ $38\overline{)950}$

$3\overline{)8 \text{ ft. and } 9 \text{ in.}}$ $5 \text{ in.}\overline{)6 \text{ ft. and } 8 \text{ in.}}$

Unit Test

1. Jack is collecting old newspapers as his way of helping to raise money for the new church building fund. One Saturday he collected 1,800 lbs. How much money did he receive at 50¢ for each 100 lbs.?

2. Fran started a bank account to help pay her way through college. She opened the account with $2.69 and put in $1.75 the next month. How much money was there in her account after she deposited these two amounts?

3. Bob, Chuck, Ed, and Lou decided to buy a box of candy that cost $1.16. If they share the cost equally, how much will each pay?

4. It takes George and Bill about 128 hours to inventory all the stock in their father's lumber yard. If they have already worked for 49 hours, about how much longer should they expect it to take?

5. The class treasury of the sixth grade was to receive 10¢ for every football ticket that the members of the class sold during the month of September. If the class sold 326 tickets, how much money went into the treasury?

6. Fred bought a bicycle so that he could give better service to the customers on his paper route. How much must he pay each week as time payments if his balance is $45.00 and he has 36 weeks in which to pay it off?

7. Add:

43	14	23	33	52	66
55	72	45	28	47	38

8. Subtract:

58	36	81	63	72	57
22	15	60	31	48	29

9. Divide: $18\overline{)72}$ $21\overline{)168}$ $51\overline{)1734}$ $68\overline{)3296}$

10. Multiply:

16	23	21	42	19	47	57	62
12	14	30	26	38	29	32	48

UNIT 2

Everyday Figuring Makes Use of Parts

When you find the how-much or the how-many of something, you are measuring; you are giving a value to something in terms of some other fixed thing. This fixed thing is called a unit. For example, a length of 12 ft. is a value that is given to a length that has 12 parts, each part being equal to a 1-unit thing called a foot.

Sometimes when you measure with these units, they fit exactly, that is, no part is left over. In these cases *whole numbers* tell the how-much or how-many. Then there are other times when there is a need to tell measurements in parts of the 1-unit thing. These parts are called the *fractional parts* of the unit.

Fractional parts often have names, like the inch which is $\frac{1}{12}$ of a foot or the minute which is $\frac{1}{60}$ of the hour.

How have you used fractional parts of units within the last week? Where have you used a part of a unit that had its own name? Are these parts often used as units by themselves? Name some that you know.

The numbers which tell the sizes of fractional parts are **fractions**. See how they are used on the opposite page.

1. Where is a whole cut into 8 equal parts? Each one of these parts is called one-eighth of a whole. The number is $\frac{1}{8}$. Are 3 of the eighth parts a fractional part of the original whole? Why?

2. What do you mean when you say that you have cut a whole into halves? into fourths? into sixths?

3. What can you call each part of the rope in the picture of the boys making jump ropes for the girls?

4. Is an inch a fractional part of a foot? Is an inch also a fractional part of a yard? Why?

5. Is a quarter a fractional part of one dollar? What is three fourths of one dollar?

6. How are fractions used in cooking?

7. The girls in the picture are cutting the piece of cardboard into how many equal pieces? What is the fraction that tells the size of one part?

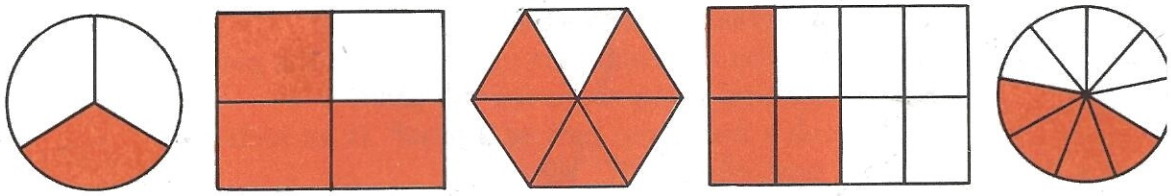
Parts Related to Wholes

As a prize for selling the most cookies in a recent sale, the girl scouts of the Beaver Patrol were given a cake. There were 8 girls in the patrol, so they needed to divide the cake into 8 parts. They knew that each part would be one-eighth. Five of the 8 girls saw how the cake was cut. They could take only 5 of the eighths of the cake. They saved the rest for the other 3 girls.

The part of the cake that was theirs can be written as a fraction $\frac{5}{8}$. It is read five-eighths. The 8 is the **denominator**; it tells the number of equal parts into which the whole has been divided. The 5 is the **numerator**; it tells how many of the 8 parts are used.

1. A whole is to be divided into 5 equal parts and 2 of them are to be used. How would you write this as a fraction? Which is the numerator? the denominator?

2. Write fractions to stand for the following drawings (the shaded parts are the parts being used):



3. Tell the meanings of the following fractions:

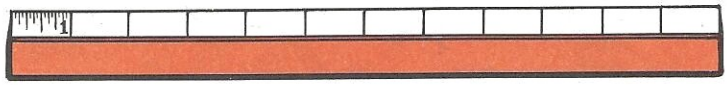
$\frac{1}{8}$ $\frac{3}{9}$ $\frac{5}{7}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{2}{3}$ $\frac{4}{5}$ $\frac{7}{12}$ $\frac{9}{11}$

4. How many parts are being used in each of the following fractions?

$\frac{2}{3}$ $\frac{7}{11}$ $\frac{4}{9}$ $\frac{3}{5}$ $\frac{5}{7}$ $\frac{11}{12}$ $\frac{3}{4}$ $\frac{6}{13}$ $\frac{1}{2}$ $\frac{5}{6}$ $\frac{7}{8}$

The Measurement Idea of a Fraction

You can think of a fraction as a value obtained by a measuring process.



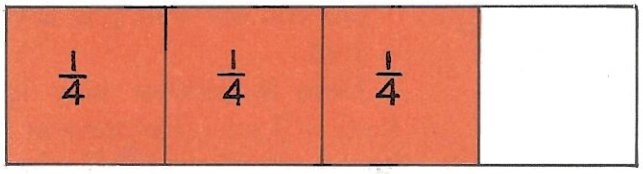
Look at the ruler at the right. You know that you can use the part 1 in. as a unit to measure things. You also know that the inch is a twelfth of a foot ($\frac{1}{12}$ ft.).

Suppose you wish to measure the length shown in this picture. If you use the unit called the inch, its length is 3 in., a whole number.

Now suppose that you do not want to use the unit name of inch for the units in the measurement. Instead, choose the fractional part of the foot ($\frac{1}{12}$ ft.) to identify your unit. Then the length is 3 of the $\frac{1}{12}$ ft. units or $\frac{3}{12}$ ft.

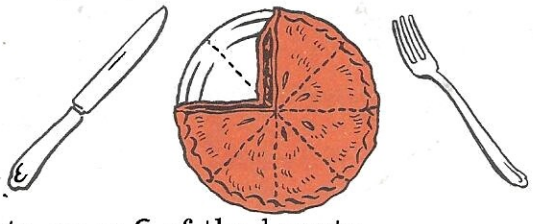
You can even think this way:

1 in. + 1 in. + 1 in. = $\frac{1}{12}$ ft. + $\frac{1}{12}$ ft. + $\frac{1}{12}$ ft. = $\frac{3}{12}$ ft. when adding the number of parts of twelfths that are being grouped together.



Look at the rectangle on the right. The shaded part is $\frac{3}{4}$ of the whole because 3 of the 4 equal parts are used.

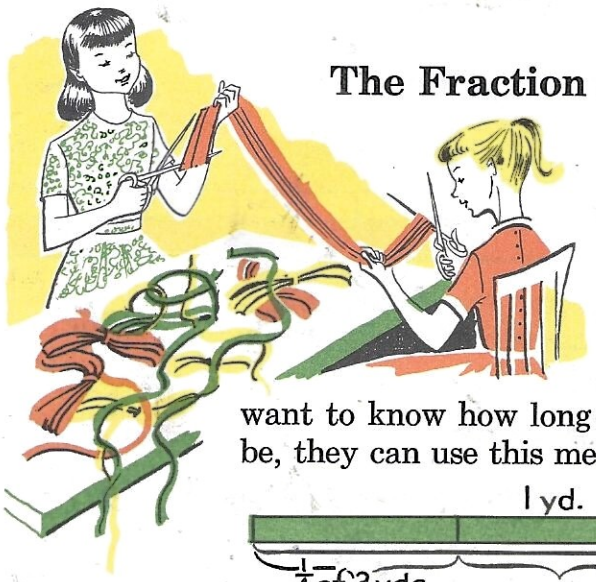
Also, if you consider the part as a $\frac{1}{4}$ unit, the value $\frac{3}{4}$ is 3 of the $\frac{1}{4}$ units.



1. In this picture you see $\frac{3}{4}$ of a pie. Tell how it may be considered as 3 of the $\frac{1}{4}$ parts, or as 6 of the $\frac{1}{8}$ parts.

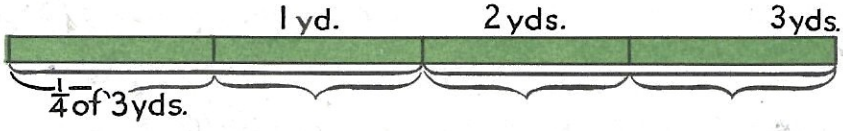
2. How many of the fractional units are found in each of these values? What is the size of the fractional unit in each case?

$\frac{4}{7}$ $\frac{5}{12}$ $\frac{9}{16}$ $\frac{7}{10}$ $\frac{5}{6}$ $\frac{8}{9}$ $\frac{3}{11}$ $\frac{8}{5}$ $\frac{6}{4}$ $\frac{7}{3}$

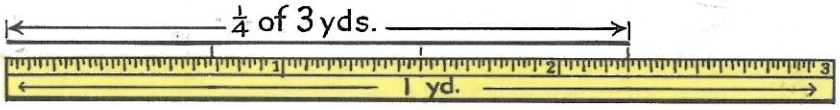


The Fraction as an Indicated Division

These two girls have several pieces of ribbon, each 3 yd. long. They want to divide them into equal parts to make streamers for a parade. They can fold the ribbon and cut it as shown in the picture. If they want to know how long each piece for the streamers will be, they can use this method:

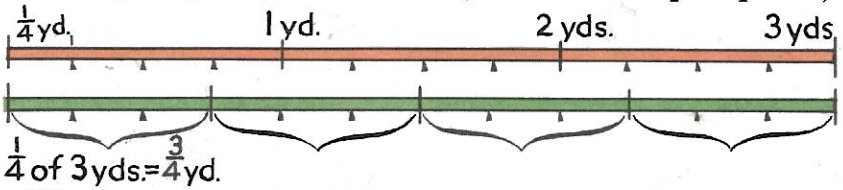


Now, if they measure the piece that is $\frac{1}{4}$ of 3 yd. with a yard stick, they will have this:



Do you see that $\frac{1}{4}$ of 3 yd. is $\frac{3}{4}$ of 1 yd.? So, 3 yd. $\div$ 4 = $\frac{3}{4}$ yd.

Here is another way to show that $\frac{1}{4}$ of 3 yd. = $\frac{3}{4}$ of 1 yd. = $\frac{3}{4}$ yd. (Before you try to divide the group of 3 yd. into 4 equal parts, divide each yard into 4 equal parts.)



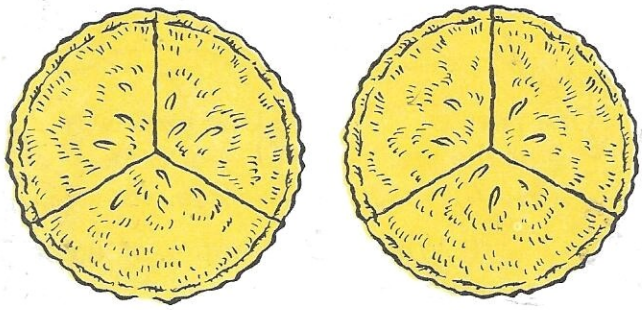
Why is each small unit equal to $\frac{1}{4}$ yd.? How many $\frac{1}{4}$ units are there in the 3 yd.?

12 small units are now equal to the 3 large units: 12 fourths = 3 wholes. So the 12 units divided into 4 equal parts is $4\overline{)12}$ = 3 units. Since each unit is $\frac{1}{4}$ yd., the 3 units are $\frac{3}{4}$ yd.

When 3 yd. $\div$ 4 = $\frac{3}{4}$ yd., the fraction is being used to express a division. This idea of a fraction is sometimes called *size*. It is used when you are dividing a group of units into a number of parts.

1. Make a drawing on a sheet of paper to show that when 2 yd. is divided into 3 equal parts, each part will be $\frac{1}{3}$ of 2 yd. Then make another drawing to show that $\frac{1}{3}$ of 2 yd. = $\frac{2}{3}$ of 1 yd. = $\frac{2}{3}$ yd.

2. Sue, Faye, and Lois wanted to share 2 small pies equally. They divided each pie into 3 equal parts. If Sue takes a $\frac{1}{3}$ part of each of the 2 pies, will she have $\frac{2}{3}$ of a pie in all?



$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$. Why? If she took $\frac{2}{3}$ of one of the pies, will she have the same amount of pie? Is this the same idea as $\frac{2 \text{ pies}}{3} = \frac{2}{3}$ pie, which is a division of 2 units into 3 equal parts, or 2 $\div$ 3?

3. Draw pictures of 4 candy bars. Show by drawing lines how 5 boys could cut them up so they could be shared equally among them. What part of 1 candy bar would each boy have?

4. Suppose 5 girls want to cut a piece of ribbon into 5 equal parts. What is the size of each of these equal parts?

5. Tell how these fractions show that a group of wholes has been divided into an equal number of parts:

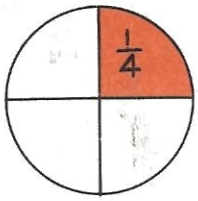
$\frac{3}{7}$ $\frac{5}{8}$ $\frac{2}{5}$ $\frac{5}{9}$ $\frac{7}{10}$ $\frac{5}{16}$ $\frac{5}{6}$ $\frac{7}{12}$ $\frac{13}{16}$

6. 3 yd. $\div$ 5 = ? 5 lb. $\div$ 8 = ? \$3.00 $\div$ 4 = ?

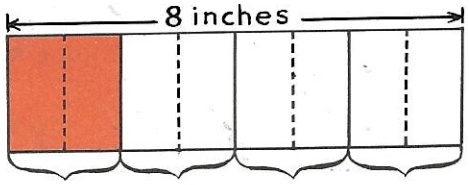
7. 4 $\div$ 5 = ? 6 $\div$ 7 = ? 4 $\div$ 9 = ? 5 $\div$ 8 = ?

Finding the Fractional Part of a Whole

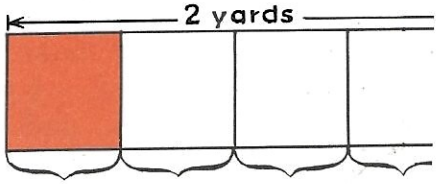
When you divide a single thing or a group of things into 4 equal parts, you can say that you have $\frac{1}{4}$ of the whole in each part.



One part is $\frac{1}{4}$ of the pie. $(1 \div 4)$



One part is $\frac{1}{4}$ of 8 in. = 2 in. $(8 \div 4)$



One part is $\frac{1}{4}$ of 2 yd. $(2 \div 4)$

1. When you want $\frac{1}{2}$ of some whole expressed as a number value, can you divide that value by 2?

2. What is $\frac{1}{2}$ of 6 lb.? Does this give you an answer that is a whole number, or is it a fraction?

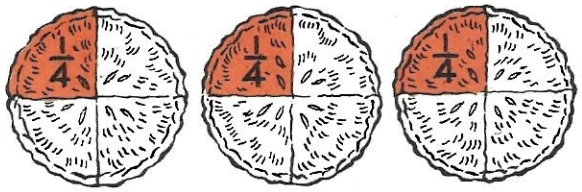
3. What is the size of the part for $\frac{1}{4}$ of 3 lb.? $\frac{1}{8}$ of 5 lb.?

4. Does $6 \div 7$ give you the same fraction as $\frac{1}{7}$ of 6? Explain.

5. Find $\frac{1}{4}$ of 56. Do you divide 56 by 4? Tell why.

6. Tell why $\frac{2}{3}$ may mean that you have taken $\frac{1}{3}$ of a group of 2 things.

7. Sam wants to use $\frac{1}{8}$ of a piece of rope that is 5 yd. long. Tell why the length can be considered either as 5 yd. $\div$ 8, or as $\frac{1}{8}$ of 5 yd.



8. $7 \div 3 = \frac{1}{3}$ of 7. Why?

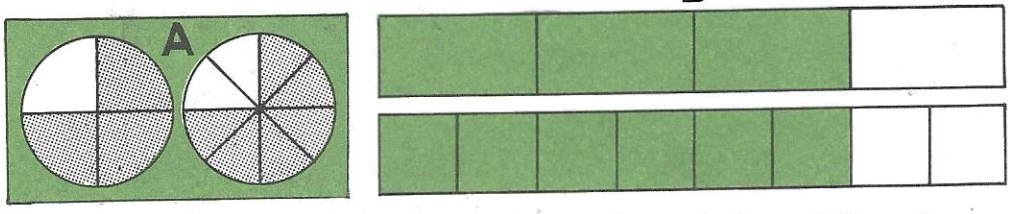
9. $\frac{1}{5}$ of $4 = 4 \div 5$. Why?

10. $\frac{1}{4}$ of 3 pies $= \frac{3}{4}$ of 1 pie. Tell how this answer can be found by considering $\frac{1}{4}$ of each of the pies in this picture and combining these parts. Tell how it can be found by considering the number of small units in all of the pies.

The Value of a Fraction

You know that the value of a fraction depends on the size of both its numerator and denominator. You need to consider them together, not separately. It is the relation of the how-many of the equal parts in use to the how-many of them in a whole that really shows the value.

Is a fraction with a numerator of 6 larger than one with a numerator of 3? Why or why not?



In diagram A, there are 3 parts of one circle and 6 parts of the other which are colored. Which circle has its larger part (the part of the whole) colored?

Why does $\frac{3}{4} = \frac{6}{8}$?

Is the same comparison true in diagram B?

An eighth ($\frac{1}{8}$) is half as large as a fourth ($\frac{1}{4}$). Why?

Do you see in the equal fractions $\frac{3}{4} = \frac{6}{8}$ that the denominator of the second is twice as large as the denominator of the first? Then why must the numerator of the second be twice as large as the numerator of the first?

If the numbers in one fraction were three times as large as those in another fraction, would the fractions have the same value? Why?

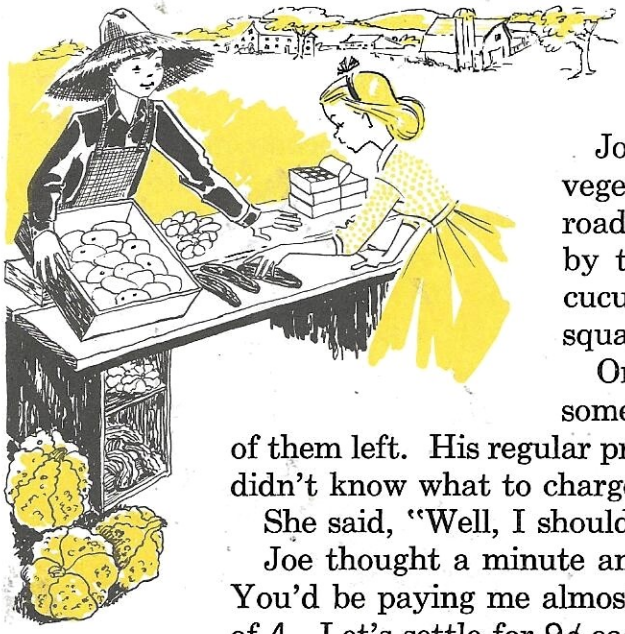
Tell why these pairs of fractions have the same value:

$\frac{2}{3} = \frac{4}{6}$ $\quad \frac{2}{5} = \frac{4}{10}$ $\quad \frac{3}{5} = \frac{9}{15}$ $\quad \frac{2}{7} = \frac{8}{28}$

Fill the blanks:

$\frac{1}{3} = \frac{?}{9}$ $\quad \frac{1}{6} = \frac{?}{12}$ $\quad \frac{4}{5} = \frac{8}{?}$ $\quad \frac{3}{7} = \frac{6}{?}$

The numerator and denominator of a fraction are often called its **terms**. Do you see that you can change the terms of a fraction and still have equal values?



Joe's Roadside Stand

Joe is a farm boy who sells vegetables, eggs, and fruit at his roadside stand. He sells potatoes by the pound, eggs by the dozen, cucumbers in groups of 4, and squash in singles and in groups.

One day Jane stopped to buy some cucumbers. Joe had only 3 of them left. His regular price on these was 4 for 35¢. He didn't know what to charge Jane for the cucumbers.

She said, "Well, I should pay you 10¢ each."

Joe thought a minute and said, "No, that's too much. You'd be paying me almost as much as I get for a group of 4. Let's settle for 9¢ each."

Do you think that his answer was about right, $3 \times 9¢ = 27¢$?

Let's study the problem:

Joe's usual price:

4 for 35¢

3 for 27¢

How would you do the figuring?

Would you find the price of 1 by comparing it with the price of 4? How?

You can make a comparison by the division process:

$$\frac{1 \text{ cucumber}}{4 \text{ cucumbers}} = \frac{1}{4}.$$ Why?

Can we say that 1 cucumber is 1 of the 4 equal parts of a group of 4 and equal to $\frac{1}{4}$ of a whole? Why?

$\frac{1}{4}$ of 35¢ = $4\overline{)35}$ = about 9¢. Why?

Will 3 cucumbers be 3 of the $\frac{1}{4}$ parts in the larger group? Why?

Also, $\frac{3 \text{ cucumbers}}{4 \text{ cucumbers}} = \frac{3}{4}$ of a group, or three things out of 4 things.

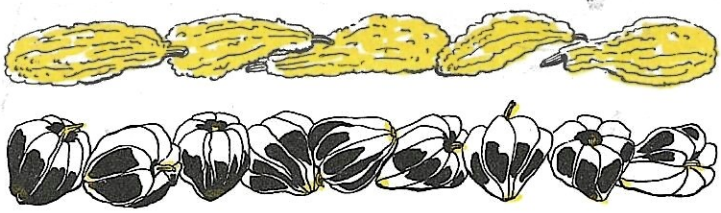
Joe really wanted to find $\frac{3}{4}$ of 35¢, didn't he? His thinking to find 27¢ must have been:

$\frac{3}{4}$ is 3 of the $\frac{1}{4}$ parts.
$\frac{1}{4}$ part of 35¢ = $4\overline{)35¢}$ = 9¢, approximately, and
3 such parts are 3 × 9¢ = 27¢.

1. What part of a group of 12 eggs is 7 eggs?

2. What fraction do you have when you compare 9 oz. of onions with a pound of onions (1 lb. = 16 oz.)?

3. Tell how a comparison of the number of summer squash with the number of acorn squash in this picture shows that there are $\frac{5}{9}$ as many summer squash as acorn.



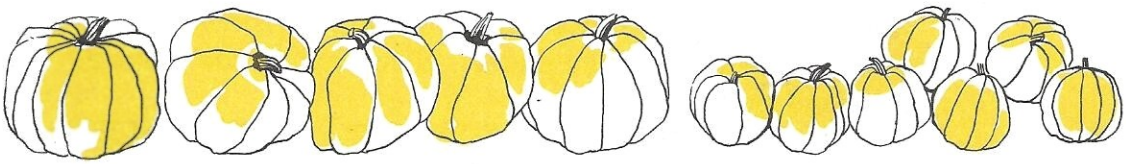
4. Look at Joe's supply of pumpkins:

a. Write a fraction that compares the number of large pumpkins with the number of small ones.

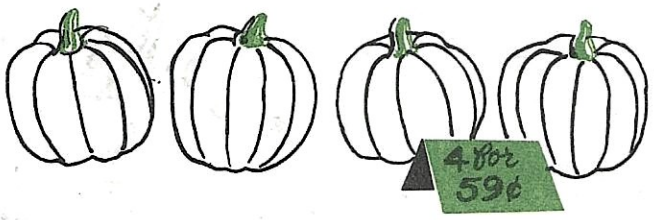
b. Write a fraction that compares the number of large ones with the total number of pumpkins.

c. Write a fraction that compares the number of small ones with the total number of pumpkins.

5. Write a fraction that compares the price of a 9¢ basket of onions with the price of a 13¢ basket.



Using What You Know About Fractions



1. Joe sold 4 pumpkins for 59¢. What should his price be for one? To solve this problem, you want to divide 59¢ into 4 equal parts. Why?

In the work shown here, the quotient is 14¢, and there is a remainder of 3¢.

$$4\overline{)59}\quad 14,\ 4,\ 19,\ 16,\ 3$$

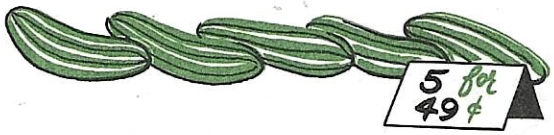
Must you divide the 3¢ to find the exact price for each pumpkin?

3¢ ÷ 4 would be the idea of a group of things divided into a number of equal parts. This would give the fractional part of $\frac{3}{4}$¢. Why?

Joe should have a price of 15¢ for 1 pumpkin. Why?

Here is another way to show the division (parts in the number 59 are used):

$4\overline{)59} = 4\overline{)56} + 4\overline{)3} = 14¢ + \frac{3}{4}¢$. In this the whole number and the fraction in the answer should be added. In cases like this, you place the two values side by side and call it addition. So, $14¢ + \frac{3}{4}¢ = 14\frac{3}{4}¢$. This result is called a **mixed number**. Can you tell why?



2. If Joe should sell 5 cucumbers for 49¢, what would be the exact price for one? What would Joe's price for 1 cucumber be?

3. How much does Joe receive for each quart basket of small white onions if he sells them at 3 qt. for 29¢?

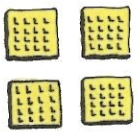
4. Write the answers to the following divisions, first as values showing the remainders, then change the answers to mixed numbers:

$3\overline{)89}$ $6\overline{)55}$ $4\overline{)99}$ $12\overline{)149}$ $22\overline{)485}$ $2\overline{)190}$ $10\overline{)203}$

Ann Likes to Make Waffles

Ann always does the cooking when the family has waffles for supper. She doesn't know how her brother Al can eat so many.

As Ann serves the waffles, she breaks them into parts which you see in this picture. What fraction tells the size of one of the parts?

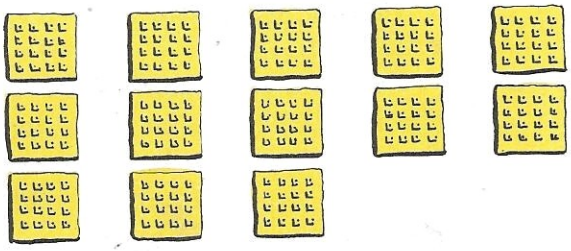


If Ann's mother ate only 3 of the parts, what part of a whole waffle did she have? Is $\frac{3}{4}$ correct? Why?

If a fraction shows that there are fewer parts in use than there are parts in the whole, it is called a **proper fraction.** $\frac{3}{5}$ and $\frac{5}{7}$ are proper fractions.

A proper fraction is one whose numerator is smaller than the denominator.

One night Ann's brother Al ate 13 of these parts of a waffle. What fraction will show the amount of waffles Al ate? Tell why $\frac{13}{4}$ is correct.



If a fraction shows that there are as many or more parts in use than there are parts in the whole, it is called an **improper fraction.** $\frac{5}{5}$ and $\frac{7}{3}$ are improper fractions.

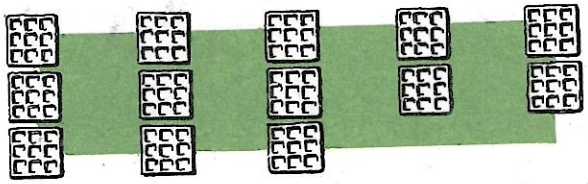
> An improper fraction is one whose numerator is equal to or larger than the denominator.

Which of these are proper and which are improper?

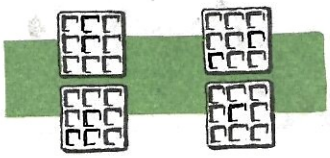
$\frac{5}{7}$ $\frac{4}{9}$ $\frac{4}{3}$ $\frac{6}{7}$ $\frac{8}{5}$ $\frac{7}{4}$ $\frac{3}{10}$ $\frac{5}{12}$ $\frac{9}{16}$ $\frac{4}{4}$

Working with Mixed Numbers

Al said, "I know of another way to tell how many waffles I ate. If I put parts together to make wholes, I see how many wholes and how many extra parts there are."



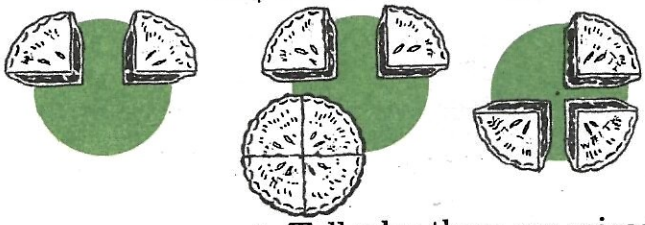
Does this picture show that Al ate 13 of the fourths of a whole waffle?



How many of the fourths does it take to make a whole? What fraction tells this amount?

Do you understand that Al ate 3 whole waffles and an extra fourth? This value is 3 and $\frac{1}{4}$. It really means $3 + \frac{1}{4}$, but it is written $3\frac{1}{4}$, which is a mixed number. You already know that a value that is a whole number and a fraction, combined by writing them side by side, is called a mixed number.

Can you see that $\frac{9}{4}$ is the same as $2\frac{1}{4}$? If it takes 4 of the fourths to make a whole, the number of wholes in 9 fourths would be the number of 4's in 9; $4\overline{)9} = 2$ groups of 4 with 1 as a remainder. The remainder of 1 has a value of $\frac{1}{4}$ of a waffle. Why?



1. What mixed number will tell the amount of pie in this diagram?

2. Tell why these are mixed numbers:

$1\frac{1}{3}$ $1\frac{3}{5}$ $2\frac{5}{6}$ $3\frac{1}{2}$ $2\frac{5}{8}$ $4\frac{2}{3}$ $7\frac{1}{2}$ $5\frac{1}{4}$ $5\frac{5}{9}$

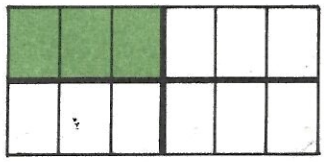
3. Change these improper fractions to mixed numbers:

$\frac{5}{3}$ $\frac{7}{6}$ $\frac{9}{5}$ $\frac{11}{4}$ $\frac{9}{7}$ $\frac{11}{3}$ $\frac{12}{5}$ $\frac{13}{6}$ $\frac{15}{8}$ $\frac{17}{4}$

Things You Should Know About Fractions

To *reduce* a fraction, you obtain smaller numbers in both the numerator and denominator. This means that you are changing to a larger part, and you will have fewer of these parts. Why?

If you change twelfths into fourths, you have 3 of the smaller parts making one of the larger. You see that $12 \div 3 = 4$ really tells the size of the larger part.



$\frac{9}{12} = \frac{?}{4}$

Since a fourth is 3 times as large as a twelfth, you will have one-third as many of them, or $9 \div 3 = 3$.

Thus, you divide both the numerator and denominator by the same number to change (or reduce) a fraction to lower terms. Dividing both by the largest number possible changes a fraction to **lowest terms.**

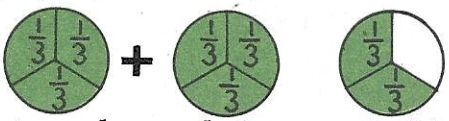
$$\frac{3\overline{)9}}{3\overline{)12}} = \frac{3}{4}$$

A *mixed* number is a number made up of one or more whole units plus a fractional part of one whole unit, like $5\frac{2}{3}$.

A *proper* fraction is less than one whole unit.

An *improper* fraction is one that has enough of the fractional parts to make a whole one or a mixed number, like $\frac{4}{4}$ or $\frac{5}{4}$.

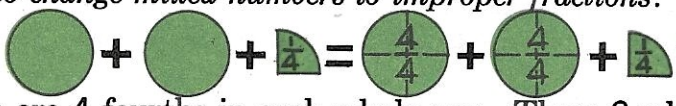
How to change improper fractions to mixed numbers:



It takes 3 thirds to make a whole one. 8 thirds = 2 whole ones $(8 \div 3)$, and 2 more thirds.

$$\frac{8}{3} = 2\frac{2}{3}$$

How to change mixed numbers to improper fractions:



There are 4 fourths in each whole one. These 2 whole ones $= 4 \times 2$, or 8 fourths. 8 fourths + 1 fourth = 9 fourths, or $\frac{8}{4} + \frac{1}{4} = \frac{9}{4}$. Thus, $(4 \times 2) + 1 = 9$ fourths.

$$2\tfrac{1}{4} = \tfrac{?}{4}$$
$$2\tfrac{1}{4} = \tfrac{9}{4}$$

Playing House

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Eight girls were playing house on the school playground. They drew a rectangular house which they divided into 8 rooms like this. Each girl's room was therefore $\frac{1}{8}$ of the house. Sue had the first room, and her sister had the second. What part of the house did the two of them have? $\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$.

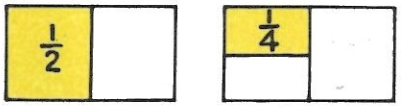
Fractions that have the same number in the denominator are called **like fractions**. This same denominator is called a **common denominator**.

> The sum of fractions having a common denominator is found by adding the numerators and placing the sum over the common denominator.

$$\begin{array}{r} \frac{2}{8} \\ +\frac{1}{8} \\ \hline \frac{3}{8} \end{array}$$

If Betty were to put her room with those of Sue and her sister, how much of the house would the 3 girls have? $\frac{1}{8} + \frac{2}{8} = ?$

Do you see that you have fractions with a common denominator? 2 of a unit + 1 of that same unit = 3 of that unit. So, $\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$.



1. The fractions $\frac{1}{4}$ and $\frac{1}{2}$ cannot be added as they now stand. Why?

2.

a.	**b.**	**c.**	**d.**	**e.**	**f.**	**g.**	**h.**	**i.**
$\frac{1}{4}$	$\frac{4}{9}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{4}{7}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{2}{4}$
$+\frac{3}{4}$	$+\frac{2}{9}$	$+\frac{2}{3}$	$+\frac{1}{5}$	$+\frac{2}{7}$	$+\frac{5}{6}$	$+\frac{4}{8}$	$+\frac{1}{6}$	$+\frac{1}{4}$

3. Add: $\frac{3}{8} + \frac{1}{8}$, $\frac{4}{7} + \frac{2}{7}$, $\frac{7}{12} + \frac{5}{12}$, $\frac{2}{9} + \frac{6}{9}$, $\frac{3}{16} + \frac{7}{16}$, $\frac{1}{10} + \frac{7}{10}$.

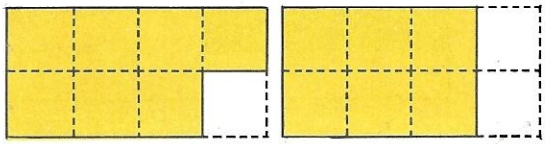
Subtracting Like Fractions

Helen was given $\frac{7}{8}$ of a yard of linen by her aunt. She decided to use part of it to make a handkerchief. If she were to use $\frac{1}{8}$ of a yard for the handkerchief, how much would she have left?

$$\begin{array}{r} \frac{7}{8} \\ -\frac{1}{8} \\ \hline \frac{6}{8} \end{array}$$

Notice the subtraction written at the left.

It can also be written like this: $\frac{7}{8} - \frac{1}{8} = \frac{6}{8}$



The difference between two fractions having a common denominator is found by subtracting the numerators and writing the result over the common denominator.

1. It was $\frac{9}{16}$ of a mile from John's home to school. Sue lived only $\frac{5}{16}$ of a mile from school. How much farther from school was John's home than Sue's?

2. Donna spent $\frac{3}{4}$ of an hour helping her mother with the housework. Sally helped her mother for $\frac{1}{4}$ of an hour. How much longer did Donna spend doing housework than Sally?

Subtract the following:

3.

a.	**b.**	**c.**	**d.**	**e.**	**f.**	**g.**	**h.**	**i.**	**j.**
$\frac{4}{5}$	$\frac{7}{12}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{8}{10}$	$\frac{7}{9}$	$\frac{2}{3}$	$\frac{7}{8}$	$\frac{3}{5}$	$\frac{7}{10}$
$\frac{2}{5}$	$\frac{5}{12}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{3}{10}$	$\frac{5}{9}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{5}$	$\frac{4}{10}$

4.

$\frac{9}{16}$	$\frac{10}{12}$	$\frac{11}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{9}{10}$	$\frac{9}{12}$	$\frac{15}{16}$	$\frac{6}{8}$	$\frac{11}{12}$
$\frac{4}{16}$	$\frac{5}{12}$	$\frac{4}{16}$	$\frac{2}{8}$	$\frac{6}{16}$	$\frac{2}{10}$	$\frac{4}{12}$	$\frac{12}{16}$	$\frac{1}{8}$	$\frac{4}{12}$

5.

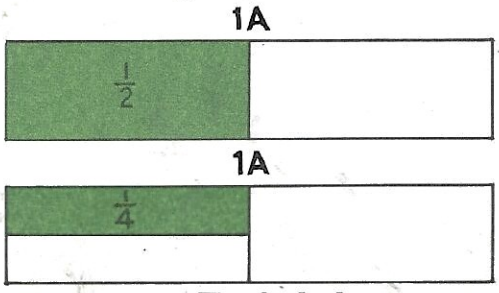
a.	**b.**	**c.**	**d.**
$\frac{3}{5} - \frac{1}{5} = ?$	$\frac{7}{8} - \frac{4}{8} = ?$	$\frac{7}{10} - \frac{4}{10} = ?$	$\frac{3}{4} - \frac{2}{4} = ?$

6. $\frac{9}{10} - \frac{6}{10} = ?$ $\quad$ $\frac{10}{12} - \frac{7}{12} = ?$ $\quad$ $\frac{5}{8} - \frac{2}{8} = ?$ $\quad$ $\frac{7}{12} - \frac{2}{12} = ?$

7. How much larger is $\frac{7}{9}$ than $\frac{2}{9}$?

8. How much larger is $\frac{12}{16}$ than $\frac{5}{16}$?

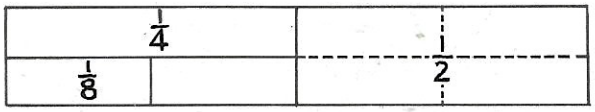
Changing Fractions to a Common Denominator



Joe had a plot of ground that was planted in potatoes. His father said it was $\frac{1}{2}$ acre ($\frac{1}{2}$ A.). He also had $\frac{1}{4}$ of an acre planted in cabbages. He wanted to know the size of the 2 plots together.

To find the answer to his problem, Joe drew pictures like the ones shown above. He thought to himself, "I can add the two fractions, $\frac{1}{2} + \frac{1}{4}$, if I can find a common denominator; they must have like parts."

Joe then thought that he could divide a whole acre into eighths or fourths.



$\frac{1}{2}$ A. = $\frac{4}{8}$ A. Why?
$\frac{1}{4}$ A. = $\frac{2}{8}$ A. Why?

If he used this size part for a common denominator, what would be the sum of the two plots?

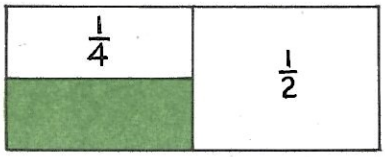
$$\frac{4}{8} + \frac{2}{8} = \frac{?}{8}$$

If he used the $\frac{1}{4}$ as the common unit or common denominator, he would have:

$\frac{1}{2}$ A. = $\frac{2}{4}$ A. Why?
$\frac{1}{4}$ A. = $\frac{1}{4}$ A.

Sum = $\frac{2}{4} + \frac{1}{4} = \frac{?}{4}$

You will find that it is always better to use the smallest common denominator. This will be the largest common unit. Why?



Which is the larger common unit, $\frac{1}{4}$ or $\frac{1}{8}$?

Can both halves and fourths be changed to sixteenths? How?

1. Find the smallest (or lowest) common denominator for these sets of fractions:

$\frac{1}{4}, \frac{3}{8}$ $\frac{1}{3}, \frac{1}{6}$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ $\frac{1}{3}, \frac{1}{4}, \frac{5}{12}$

5. Joe needs some fencing on one side of his stand. The kind that he wants is sold by the foot.

How many feet of fencing does Joe need for a space 89 in. long?

$$\begin{array}{r} 7 \\ 12\text{ in.})\overline{89\text{ in.}} \\ \underline{84} \\ 5\text{ in.} \end{array}$$

How many groups of 12 in. are there in 89 in.?

Using the remainder method, you see that Joe needs 7 ft. and 5 in.

You can express this in feet:

7 feet and 5 in. = 7 ft. and $\frac{5}{12}$ ft. = $7\frac{5}{12}$ ft.

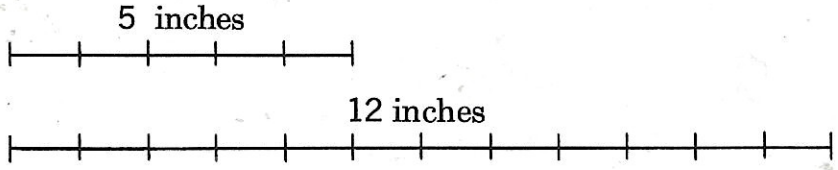
How was the $\frac{5}{12}$ ft. found?

$$\begin{array}{r} 7\frac{5}{12}\text{ ft.} \\ 12\text{ in.})\overline{89\text{ in.}} \\ \underline{84} \\ 5\text{ in.} \end{array}$$

$$\frac{5\text{ in.}}{12\text{ in.}} = \frac{5}{12}$$

a. When you compare 5 inches with 12 in., the 5 stands for the parts you have of the 12 equal parts in a foot, or $\frac{5}{12}$ ft.

b. When you compare a length of 5 in. with a length of 12 in., the shorter length has a value $\frac{5}{12}$ as great as the longer one.



6. How many groups of 8 in. are there in 99 in.?

7. How many half-dozen eggs are there in 77 eggs?

8. How many 5 lb. bags can Joe fill from 89 lb. of onions? What part of a bag will he have left over?

9. 187 eggs are how many dozen eggs?

10. Give the number of groups in each of these divisions as a mixed number:

3 ft.)17 ft. 4 in. 16 oz.)251 oz. 12 eggs)271 eggs

Practice Work

1. Tell why $\frac{3}{5}$ is a larger fractional value than $\frac{3}{7}$; why $\frac{2}{9}$ is a smaller value than $\frac{2}{7}$.

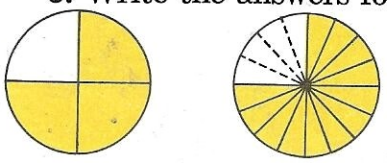
2. Which is larger, $\frac{1}{4}$ of 3 yd. or $\frac{1}{6}$ of 3 yd.? Why?

3. What fractional part tells the value of $\frac{1}{12}$ of 11?

4. Which is larger, $\frac{5}{9}$ or $\frac{7}{9}$? Tell why.

5. How many fourths are there in a whole one? How many eighths?

6. Write the answers for Ex. 5 in fraction form.



7. Use the figures shown here to tell why $\frac{3}{4}$ of a whole is the same value as $\frac{12}{16}$ of it.

8. Write a fraction that compares the value of a 1-cent coin with a 5-cent coin.

9. Write a fraction that compares a measurement of 8 ft. with one of 15 ft.

10. An 8 in. length is what fractional part of a 9 in. length?

11. Why can we say that 5 in. and 7 in. compare as 5 to 7, or $\frac{5}{7}$?

12. You have 26¢ in your pocket. You spend 15¢ out of the 26¢. What fractional part of it do you spend? What fractional part of it do you have left?

13. A hat was originally marked at \$5.00. The sale price is \$4.00. What part of the original marked price is the sale price?

14. Express the quotients for these divisions as mixed numbers:

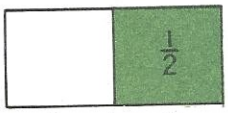
$$12\overline{)651 \text{ lb.}} \qquad 12 \text{ in.}\overline{)651 \text{ in.}} \qquad 24 \text{ hr.}\overline{)725 \text{ hr.}} \qquad 32\overline{)985 \text{ ft.}}$$

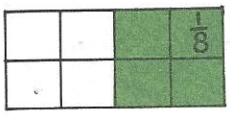
54	54		
60	60		
51	51		
48	48		
3 lb.	3		

Easy Steps in Finding the L. C. D.

Probably the easiest way to find the smallest common denominator or the L. C. D. of a group of fractions is to:

a. First examine the units in the fractions as they stand. If they are unlike, see if the largest denominator tells you that you have a unit into which the other unit or units (if there is more than one other fraction) can be changed.

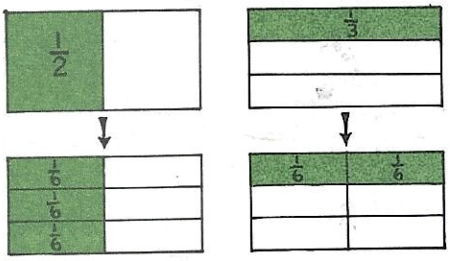




Can halves be changed into eighths? If so, the L. C. D. is *present.*

$\frac{1}{2} = \frac{4}{8}$. Why? $\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = ?$ $\frac{1}{2} + \frac{1}{4} = ?$

b. When the L. C. D. is not present, you must find it by some other method. For example, to add $\frac{1}{2}$ and $\frac{1}{3}$, change the smaller unit ($\frac{1}{3}$) into sixths, a unit half as large: $\frac{1}{3} = \frac{2}{6}$. Why?



Next check to see whether the other fraction can be changed to sixths: $\frac{1}{2} = \frac{3}{6}$. Why?

To add these fractions, you can write them this way:

$$\begin{array}{rcl} \frac{1}{2} & = & \frac{3}{6} \\ +\frac{1}{3} & + & \frac{2}{6} \end{array} \quad = \quad \frac{5}{6}$$ Why?

c. If dividing the smaller unit into 2 equal parts does not give the L. C. D., try 3, 4, or some other number of parts. You see that the division of a unit into 2 *equal parts multiplies the denominator by* 2. Why do you multiply the numerator by the same number that is used to change the size of the unit?

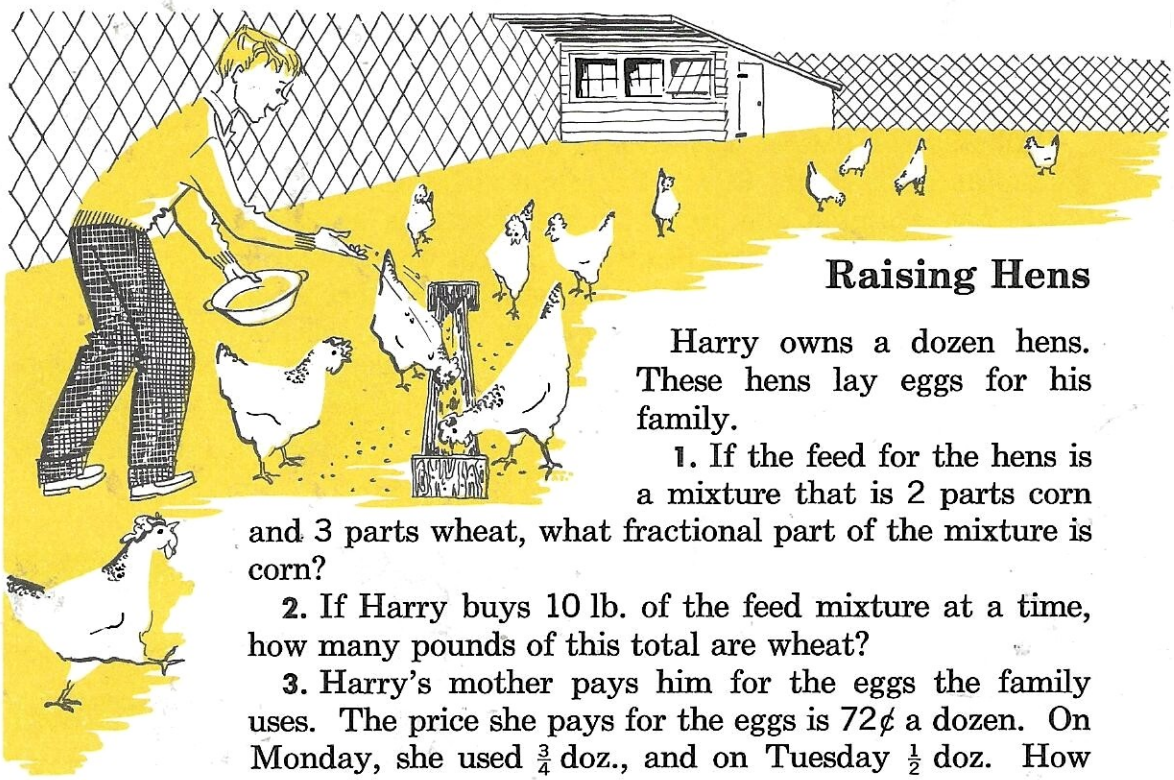
Add:

$$\begin{array}{rcl} \frac{2}{3} & = & \frac{?}{12} \\ +\frac{1}{4} & + & \frac{?}{12} \\ \hline ? & & ? \end{array}$$

The smaller unit is the fourth. Change $\frac{1}{4}$ to eighths: $\frac{1}{4} = \frac{2}{8}$. Why? Can you change thirds to eighths?

Try changing fourths into 3 equal parts: $\frac{1}{4} = \frac{?}{12}$. Can you change thirds to twelfths?

Finish the work.



Raising Hens

Harry owns a dozen hens. These hens lay eggs for his family.

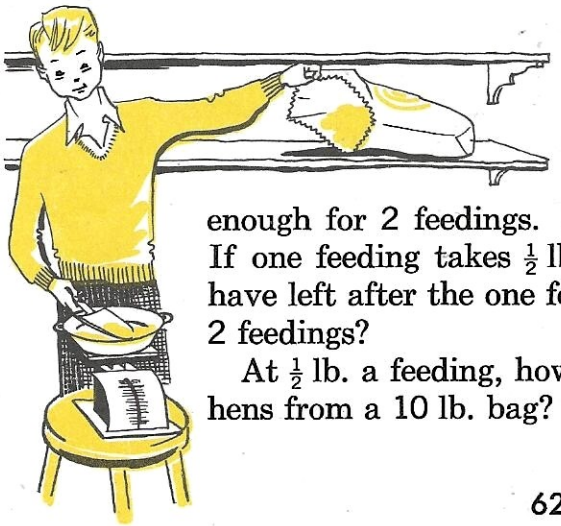
1. If the feed for the hens is a mixture that is 2 parts corn and 3 parts wheat, what fractional part of the mixture is corn?

2. If Harry buys 10 lb. of the feed mixture at a time, how many pounds of this total are wheat?

3. Harry's mother pays him for the eggs the family uses. The price she pays for the eggs is 72¢ a dozen. On Monday, she used $\frac{3}{4}$ doz., and on Tuesday $\frac{1}{2}$ doz. How many dozen eggs did she use altogether?

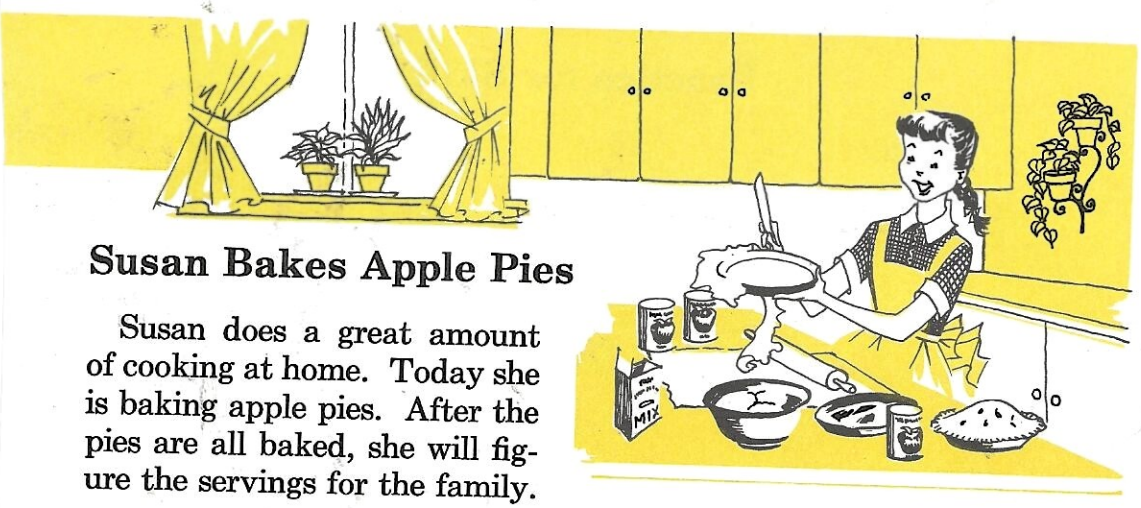
4. What did Harry's mother pay him for the eggs she used on Monday and Tuesday?

5. What part of a dozen is 4 eggs? 10 eggs? 8 eggs?



Harry always weighs the feed that he gives his hens. Once, when his supply was getting low, he decided to see if he had enough for 2 feedings. He found that he had $\frac{7}{8}$ lb. left. If one feeding takes $\frac{1}{2}$ lb., what part of a pound will he have left after the one feeding? Does he have enough for 2 feedings?

At $\frac{1}{2}$ lb. a feeding, how many times can Harry feed his hens from a 10 lb. bag?



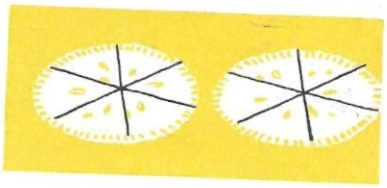
Susan Bakes Apple Pies

Susan does a great amount of cooking at home. Today she is baking apple pies. After the pies are all baked, she will figure the servings for the family.

1. If she bakes 3 pies and cuts them into sixths, what part of 1 pie will serve 4 people if each takes only 1 piece?

2. How many $\frac{2}{3}$'s parts are there in 2 whole pies?

3. If Susan's family eats only $\frac{2}{3}$ of 1 pie as dessert for 1 dinner, for how many dinners would she have dessert from 2 whole pies?



4. Susan gives $\frac{1}{6}$ of a pie to a friend. This leaves her $\frac{5}{6}$ of a pie. If $\frac{2}{3}$ of this same pie is served at dinner, what part of a pie is left for another meal?

5. If Susan has $\frac{1}{6}$ of one pie and $\frac{1}{2}$ of another, what part of a whole pie does she have altogether?

6. One night Susan's brother was very hungry for pie. He said, "I can eat $\frac{2}{3}$ of all of the pie you have." If there were 15 pieces in all, how many of them must her brother eat if he eats as much as he says he can?

7. If Susan cuts 3 pies into 18 equal parts and 15 of them are eaten, what part of the whole 18 pieces is not eaten?

8. If it takes 4 oz. of flour for 1 of Susan's pies, how many pies could she bake with 34 oz. of flour?

Practice for Skill

1. Add:

a.	b.	c.	d.	e.
$\frac{1}{2} + \frac{1}{4}$	$\frac{1}{3} + \frac{1}{6}$	$\frac{1}{4} + \frac{1}{8}$	$\frac{1}{2} + \frac{1}{6}$	$\frac{5}{12} + \frac{1}{6}$
f.	**g.**	**h.**	**i.**	**j.**
$\frac{2}{3} + \frac{5}{6}$	$\frac{3}{4} + \frac{3}{8}$	$\frac{1}{4} + \frac{5}{8}$	$\frac{1}{2} + \frac{7}{8}$	$\frac{1}{2} + \frac{3}{10}$
k.	**l.**	**m.**	**n.**	**o.**
$\frac{7}{8}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{2}{7}$
$+ \frac{3}{4}$	$+ \frac{7}{12}$	$+ \frac{5}{9}$	$+ \frac{1}{10}$	$+ \frac{3}{14}$

2. Change to improper fractions and add:

a. $1\frac{2}{3} = 1\frac{4}{6} = \frac{10}{6} = \frac{?}{6} = ?$, as a mixed number.

$2\frac{1}{6} \quad 2\frac{1}{6} \quad \frac{13}{6}$

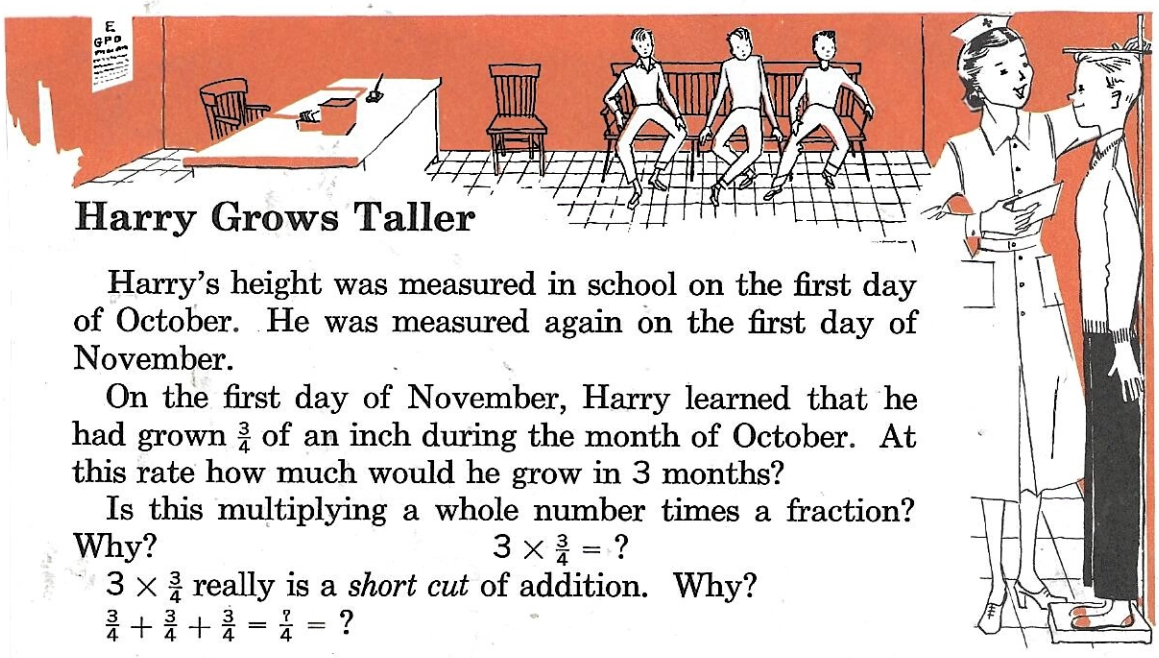
b.	c.	d.	e.	f.
$2\frac{1}{2}$	$2\frac{1}{3}$	$3\frac{1}{12}$	$2\frac{1}{5}$	$4\frac{1}{3}$
$3\frac{1}{4}$	$1\frac{1}{6}$	$1\frac{2}{3}$	$1\frac{3}{10}$	$2\frac{2}{9}$

3. Add these mixed numbers without changing them to improper fractions:

a.	b.	c.	d.
$2\frac{5}{8} = 2\frac{5}{8}$	$4\frac{4}{9}$	$5\frac{5}{6}$	$4\frac{1}{2}$
$+ 1\frac{3}{4} \quad 1\frac{6}{8}$	$3\frac{1}{3}$	$3\frac{2}{3}$	$5\frac{3}{8}$
$3\frac{11}{8} = 4\frac{3}{8}$ Why?			

4. Subtract:

a.	b.	c.	d.	e.
$\frac{3}{4} - \frac{1}{2}$	$\frac{7}{8} - \frac{3}{4}$	$\frac{5}{9} - \frac{1}{3}$	$\frac{7}{12} - \frac{1}{3}$	$\frac{4}{5} - \frac{3}{10}$
f.	**g.**	**h.**	**i.**	**j.**
$4\frac{2}{3}$	$5\frac{3}{4}$	$4\frac{5}{9}$	$4\frac{11}{12}$	$5\frac{3}{4}$
$2\frac{1}{6}$	$2\frac{1}{2}$	$1\frac{1}{3}$	$2\frac{2}{3}$	$2\frac{1}{12}$



Harry Grows Taller

Harry's height was measured in school on the first day of October. He was measured again on the first day of November.

On the first day of November, Harry learned that he had grown $\frac{3}{4}$ of an inch during the month of October. At this rate how much would he grow in 3 months?

Is this multiplying a whole number times a fraction? Why? $3 \times \frac{3}{4} = ?$

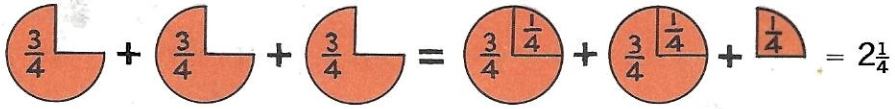
$3 \times \frac{3}{4}$ really is a *short cut* of addition. Why?

$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{?}{4} = ?$

$$\begin{array}{r} \frac{3}{4} \\ \frac{3}{4} \\ +\ \frac{3}{4} \\ \hline \end{array}$$

The units or parts are alike. What are they? You add the how-many of them: $3 + 3 + 3 = 9$. You get 9 of the unit you are using (fourths in this case). $\frac{9}{4} = \frac{8}{4} + \frac{1}{4} = 2 + \frac{1}{4}$, or $2\frac{1}{4}$. Explain.

Here is the addition, using diagrams:

$3 \times \frac{3}{4} =$  $= 2\frac{1}{4}$

The multiplication is shorter and is done this way:

$$3 \times \frac{3}{4} = \frac{3 \times 3}{4} = \frac{9}{4} = 2\frac{1}{4}$$

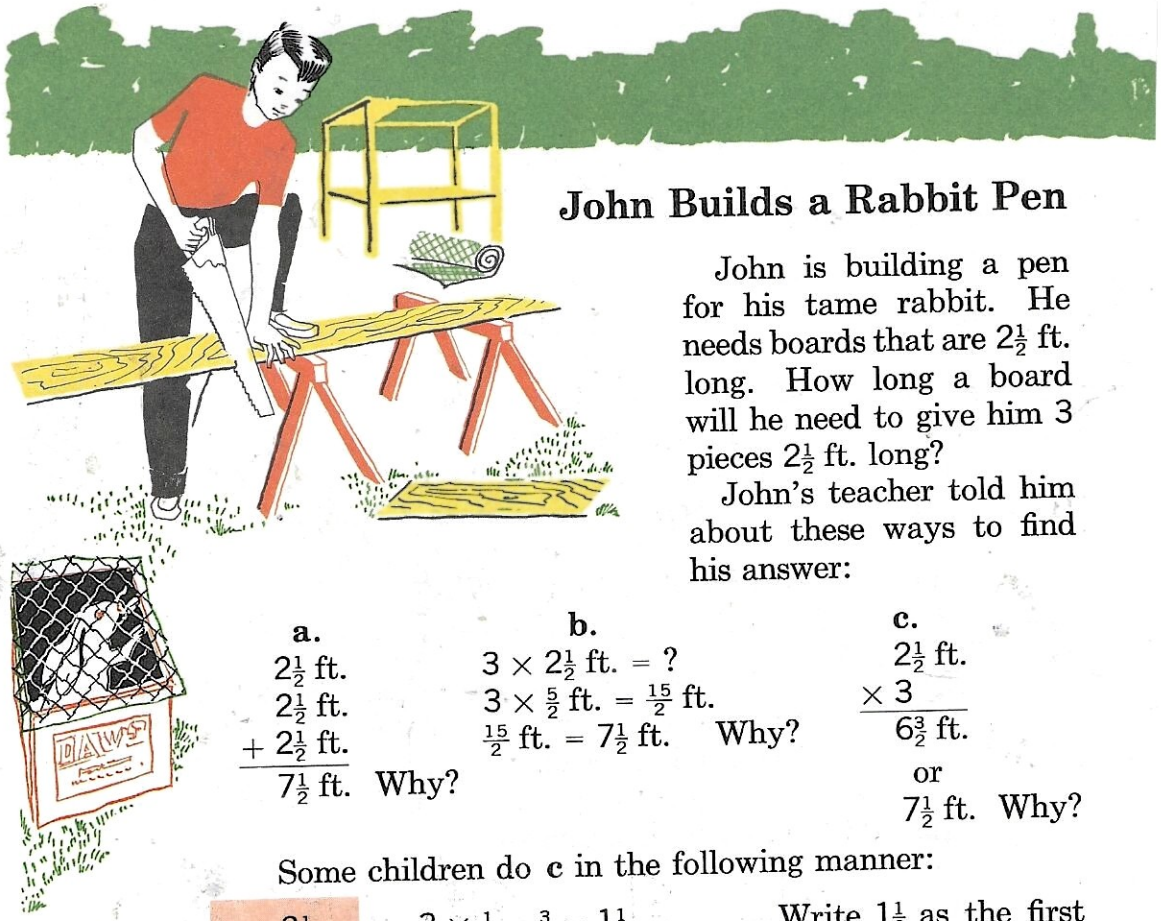
3 of the units of the $\frac{1}{4}$ size have been used 3 times: $3 \times 3 = 9$. Since it takes 4 of the $\frac{1}{4}$'s to make a whole, 9 of them would be 2 wholes and $\frac{1}{4}$ more.

1. If Harry grows $\frac{3}{8}$ of an inch in 2 weeks, how much would he grow at this rate in 4 months?

2. Multiply:

$3 \times \frac{5}{8}$ in. $= ?$ $\quad 5 \times \frac{3}{4}$ lb. $= ?$ $\quad 7 \times \frac{2}{3}$ yd. $= ?$

$5 \times \frac{3}{4} = ?$ $\quad 4 \times \frac{2}{5} = ?$



John Builds a Rabbit Pen

John is building a pen for his tame rabbit. He needs boards that are $2\frac{1}{2}$ ft. long. How long a board will he need to give him 3 pieces $2\frac{1}{2}$ ft. long?

John's teacher told him about these ways to find his answer:

a.

$2\frac{1}{2}$ ft.
$2\frac{1}{2}$ ft.
$+\ 2\frac{1}{2}$ ft.
$7\frac{1}{2}$ ft. Why?

b.

$3 \times 2\frac{1}{2}$ ft. = ?
$3 \times \frac{5}{2}$ ft. = $\frac{15}{2}$ ft.
$\frac{15}{2}$ ft. = $7\frac{1}{2}$ ft. Why?

c.

$2\frac{1}{2}$ ft.
$\times\ 3$
$6\frac{3}{2}$ ft.
or
$7\frac{1}{2}$ ft. Why?

Some children do **c** in the following manner:

$2\frac{1}{2}$
$\times\ 3$
$1\frac{1}{2}$
6
$7\frac{1}{2}$

$3 \times \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}$ Write $1\frac{1}{2}$ as the first partial product.

$3 \times 2 = 6$ Write 6 as the second partial product.

The sum of the partial products is $7\frac{1}{2}$.

1. Why is the work correct for each of these multiplications?

a.

$1\frac{3}{4}$
$\times\ 8$
6
8
14

b.

$2\frac{3}{5}$
$\times\ 5$
3
10
13

c.

$3\frac{2}{7}$
$\times\ 5$
$1\frac{3}{7}$
15
$16\frac{3}{7}$

d.

$5\frac{3}{8}$
$\times\ 7$
$35\frac{21}{8}$
or
$37\frac{5}{8}$

e.

$1\frac{5}{12}$
$\times\ 4$
$1\frac{2}{3}$
4
$5\frac{2}{3}$

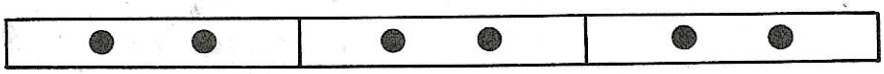
f.

$5\frac{2}{3}$
$\times\ 4$
$22\frac{2}{3}$

Gary Shares His Apples

Gary had 6 apples which he planned to share with Linda and Sue. He asked Linda to find $\frac{2}{3}$ of 6 for both Sue's and her share. The 2 shares together were $\frac{2}{3}$ of the total. Why?

When you want a fractional part of a group like $\frac{2}{3}$ of 6, it can be written $\frac{2}{3} \times 6$. *Of* means *times*:



$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

$\frac{2}{3} \times 6 = ?$
$\frac{1}{3} \times 6 = 6 \div 3 = 2$
$\frac{2}{3} \times 6 = 2 \times 2 = 4$

$\frac{2}{3} \times 6$ means $\frac{2}{3}$ of 6. When you want $\frac{2}{3}$ of something, you find $\frac{1}{3}$ of it first. $\frac{1}{3}$ of a value is found by dividing it by 3. Then $\frac{2}{3}$ would be twice this $\frac{1}{3}$ part.

There are other ways by which this multiplication can be done. For example, you can do the division that gives the $\frac{1}{3}$ part this way:

$\frac{2}{3} \times \overset{2}{\cancel{6}}$

Cross out the 3 and the 6, and write the 2 above the 6 ($6 \div 3 = 2$).

$\frac{2}{3} \times \overset{2}{\cancel{6}} = 2 \times 2 = 4$

The crossing out shows that you did the division to find $\frac{1}{3}$ in your head. Then, to get $\frac{2}{3}$, multiply 2×2.

Linda said that she could find the answer by multiplying the group by 2 and then finding $\frac{1}{3}$ of this product. Is such a process correct?

$\frac{2}{3} \times 6 = \frac{2 \times 6}{3} = \frac{12}{3} = 4.$ Why?

1. Check this example:

$\frac{3}{\cancel{4}} \times \overset{3}{\cancel{12}} = 3 \times 3 = 9$ or $\frac{3}{4} \times 12 = \frac{3 \times 12}{4} = \frac{36}{4} = 9$

Practice Work

1. Gary says that $\frac{5}{6}$ of 48 pieces of candy is 40 pieces. Find $\frac{5}{6} \times 48$ to show that he is right.

2. How many apples are there in $\frac{3}{4}$ of 2 doz.?

3. If you had a rope that was 16 ft. long and you used $\frac{3}{4}$ of it for a jump-rope, how long a piece did you use?

4. Sally had a piece of cloth that was 27 in. long. If she used $\frac{2}{3}$ of it to make a runner for her dressing table, how long a piece did she use?

5. Sue had 96¢. She decided to spend $\frac{5}{8}$ of it and save the rest. How much did she decide to save? $\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$, the fractional part to be saved. Why? $\frac{3}{8} \times 96 = ?$

6. $\frac{3}{8}$ of $32 = \frac{3}{8} \times 32 = ?$

7. $\frac{3}{5} \times 15 = ?$	$\frac{5}{8} \times 40 = ?$	$\frac{3}{7} \times 28 = ?$
8. $\frac{5}{9} \times 45 = ?$	$\frac{5}{6} \times 72 = ?$	$\frac{3}{4} \times 144 = ?$
9. $\frac{7}{9} \times 36 = ?$	$\frac{5}{9} \times 63 = ?$	$\frac{7}{8} \times 72 = ?$
10. $\frac{5}{8} \times 56 = ?$	$\frac{3}{7} \times 49 = ?$	$\frac{3}{5} \times 40 = ?$

11. $\frac{3}{4} \times 21 = \frac{3 \times 21}{4} = \frac{63}{4} = ?$

12. $\frac{2}{3} \times 16 = ?$	$\frac{3}{5} \times 18 = ?$	$\frac{2}{7} \times 40 = ?$
13. $\frac{5}{6} \times 32 = ?$	$\frac{4}{5} \times 24 = ?$	$\frac{3}{4} \times 30 = ?$
14. $\frac{5}{9} \times 8 = ?$	$\frac{7}{8} \times 30 = ?$	$\frac{7}{10} \times 9 = ?$

15. When you want to find $\frac{3}{4}$ of a number (or $\frac{3}{4} \times$ a number), you divide that number by 4 and then multiply the quotient by 3. Tell why.

16. Why will $\frac{4}{5}$ of a number give a value that is twice as large as $\frac{2}{5}$ of it?

17. Why will $\frac{3}{4}$ of a number give a value that is twice as large as $\frac{3}{8}$ of it?

18. Which gives the larger value, $\frac{3}{5}$ or $\frac{3}{7}$ of 28? How much larger?

Ann Makes a New Skirt

Ann needed a new skirt. She saw some flowered cotton material which sold for 84¢ a yard. If she decided to buy $3\frac{3}{4}$ yd., how much must she pay for the material?

This is how Ann found the cost:

$$\begin{array}{r} 84¢ \\ \times\ 3\frac{3}{4} \\ \hline 63 \\ 252 \\ \hline 315¢ \\ \text{or} \\ \$3.15 \end{array}$$

Use $3\frac{3}{4}$ in parts;
multiply 84 first by $\frac{3}{4}$, then by 3:
$\frac{3}{4} \times 84¢ = 63¢$. Why?
$3 \times 84¢ = 252¢$. Why?
Add the partial products.
The answer is 315¢ or \$3.15.

How was the form of writing the product changed from cents to dollars and cents?

1. Could Ann have done the multiplication $3\frac{3}{4} \times 84¢$ using $\frac{15}{4} \times 84¢$? See if you get the same answer.

2. What would $3\frac{7}{8}$ yd. of goods have cost Ann at a price of 56¢ a yard?

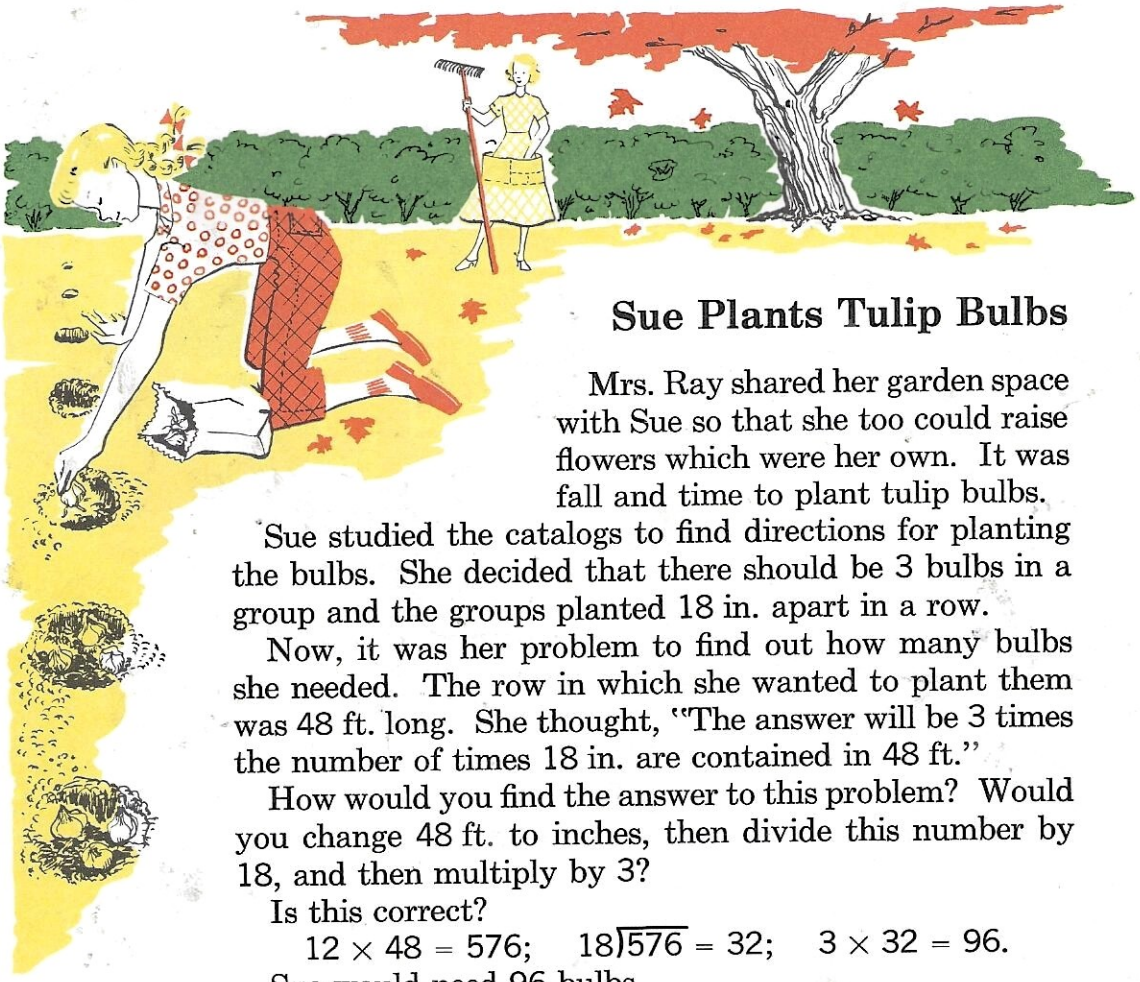
3. Find the cost of $5\frac{5}{8}$ yd. of ribbon at 24¢ a yard.

4. Multiply:

50¢	54¢	64¢	72¢	\$.48	\$1.44
$2\frac{1}{2}$	$2\frac{2}{3}$	$3\frac{3}{8}$	$4\frac{5}{6}$	$3\frac{3}{4}$	$2\frac{3}{8}$

5. Multiply by changing the mixed number to an improper fraction:

40	63	57	42	91	72
$2\frac{1}{2}$	$3\frac{3}{9}$	$2\frac{2}{3}$	$3\frac{2}{7}$	$1\frac{3}{7}$	$2\frac{5}{9}$



Sue Plants Tulip Bulbs

Mrs. Ray shared her garden space with Sue so that she too could raise flowers which were her own. It was fall and time to plant tulip bulbs.

Sue studied the catalogs to find directions for planting the bulbs. She decided that there should be 3 bulbs in a group and the groups planted 18 in. apart in a row.

Now, it was her problem to find out how many bulbs she needed. The row in which she wanted to plant them was 48 ft. long. She thought, "The answer will be 3 times the number of times 18 in. are contained in 48 ft."

How would you find the answer to this problem? Would you change 48 ft. to inches, then divide this number by 18, and then multiply by 3?

Is this correct?

$12 \times 48 = 576$; $18\overline{)576} = 32$; $3 \times 32 = 96$.

Sue would need 96 bulbs.

She solved the problem another way.

Here is her work:

18 in. $= \frac{1}{2}$ yd. 48 ft. = 16 yd.

16 yd. $\div \frac{1}{2}$ yd. = ?

1 yd. $\div \frac{1}{2}$ yd. = 2

In 16 yd. I have

$16 \times 2 = 32$

There would be 32 groups of 3 bulbs each, or $32 \times 3 = 96$ bulbs.

She thought, "How many half-yards in 1 yd.? In 16 yd. there would be 16 times as many as there are in 1 yard."

Was her thinking correct?

Could Sue have solved her problem this way?

18 in. = $1\frac{1}{2}$ ft. Why? 48 ft. $\div$ $1\frac{1}{2}$ = ? groups of 3 bulbs each.

$48 \div 1\frac{1}{2} = 48 \div \frac{3}{2}$. Why?

$48 \div \frac{3}{2} = ?$

$48 \div \frac{1}{2} = 96$

$\frac{1}{3}$ of $96 = \frac{1}{3} \times 96 = 32$

32 groups of 3 bulbs each = 96 bulbs

Think this way:

How many halves in 1? There are 2.

How many halves in 48? There are 48 times as many as there are in 1, or $48 \times 2 = 96$.

But, it was how many $\frac{3}{2}$ in 48 that was wanted. $\frac{3}{2}$ is $3 \times \frac{1}{2}$, so the 96 halves is 3 times too great. You need $\frac{1}{3}$ of 96. $\frac{1}{3} \times 96 = 32$ groups.

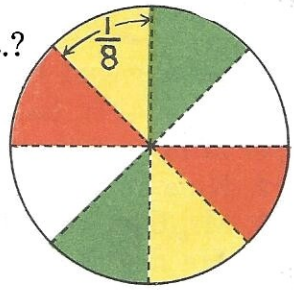
1. Does the division, $3 \div \frac{1}{2} = ?$, ask you to find how many $\frac{1}{2}$'s there are in 3? What is the answer?

2. How many $\frac{1}{4}$'s are there in 1? in 5? in 10? in 36?

3. What are you asked to find in the division $8 \div \frac{1}{3} = ?$ What is the answer?

4. How many $\frac{1}{2}$ ft. lengths are there in 16 ft.?

5. How many $\frac{1}{8}$'s are there in 1 whole? in 2 wholes? in 5 wholes? in 12 wholes?



6. How many quarters are there in $2.00? ($2 \div \frac{1}{4} = ?$)

7. Write a division that will tell you how to find the number of quarter-pounds of cheese there are in a 12 lb. block.

8. Suppose Sue had wanted to plant her bulbs 27 in. apart in a row that was 48 ft. long. How many groups of 3 would she need? Tell how you can solve this problem using the division $16 \div \frac{3}{4}$.

9. $4 \div \frac{1}{5} = ?$ $9 \div \frac{2}{3} = ?$ $7 \div \frac{1}{5} = ?$ $10 \div \frac{2}{5} = ?$

Unit Review

1. Addition:

$\frac{1}{2} + \frac{1}{3}$ $\frac{5}{6} + \frac{1}{6}$ $\frac{5}{12} + \frac{1}{4}$

2. Subtraction:

$\frac{7}{8} - \frac{3}{8}$ $\frac{5}{6} - \frac{1}{2}$ $\frac{3}{4} - \frac{2}{3}$

3. Multiplication:

$6 \times \frac{2}{3}$ $\frac{1}{2} \times 12$ $\frac{2}{3} \times 9$ $3 \times \frac{1}{5}$ $2 \times \frac{3}{5}$ $8 \times \frac{3}{4}$

4. Changing fractions to lowest terms:

$\frac{4}{12} = \frac{?}{3}$ $\frac{9}{15} = \frac{3}{?}$ $\frac{12}{16} = \frac{?}{?}$ $\frac{12}{18} = \frac{?}{?}$ $\frac{15}{21} = \frac{?}{?}$ $\frac{24}{32} = \frac{?}{?}$

5. Changing fractions to higher terms:

$\frac{1}{3} = \frac{?}{9}$ $\frac{2}{5} = \frac{?}{10}$ $\frac{3}{4} = \frac{?}{16}$ $\frac{5}{8} = \frac{15}{?}$ $\frac{3}{7} = \frac{9}{?}$ $\frac{8}{12} = \frac{?}{24}$

6. Changing mixed numbers to improper fractions:

$3\frac{1}{2}$ $4\frac{1}{3}$ $2\frac{2}{5}$ $5\frac{3}{4}$ $3\frac{5}{8}$ $1\frac{1}{3}$ $2\frac{1}{5}$ $2\frac{15}{6}$

7. Changing improper fractions to mixed numbers:

$\frac{4}{3}$ $\frac{15}{2}$ $\frac{9}{4}$ $\frac{12}{8}$ $\frac{25}{8}$ $\frac{35}{16}$ $\frac{6}{5}$ $\frac{13}{10}$ $\frac{16}{12}$

8. Add these:

$3\frac{3}{4}$ $5\frac{2}{3}$ $4\frac{3}{8}$

2 $6\frac{2}{3}$ $3\frac{1}{4}$

9. Subtract these:

$5\frac{3}{4}$ $4\frac{5}{8}$ $7\frac{3}{4}$

2 $2\frac{3}{8}$ $2\frac{1}{2}$

10. Multiply:

$3\frac{1}{2}$ $3\frac{1}{8}$ $4\frac{1}{4}$ $6\frac{2}{3}$ $5\frac{5}{8}$ $2\frac{5}{6}$ $3\frac{2}{5}$ $4\frac{3}{8}$

4 5 2 4 4 3 5 7

11. Johnny and Bill joined together 3 pieces of roofing paper, each $4\frac{1}{2}$ ft. long. What was the entire length of the paper after putting it together?

12. Nine inches is what fractional part of a yard?

13. Mr. Gregory told Bill to cut a piece of lumber $1\frac{1}{2}$ ft. long. He cut the lumber 20 in. long. The extra inches that Bill cut were what fractional part of a foot?

14. $6 \div \frac{1}{4} = ?$ $3 \div \frac{1}{6} = ?$ $4 \div \frac{2}{3} = ?$ $9 \div \frac{3}{10} = ?$

15. Tell how you would check these:

a. $\frac{2}{3} + \frac{3}{4} = 1\frac{5}{12}$

b. $2\frac{1}{4} - 1\frac{3}{8} = \frac{7}{8}$

c. $2 \times 3\frac{3}{5} = 7\frac{1}{5}$

16. Add:

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{3}{5}$ $\frac{2}{3}$

$\frac{3}{4}$ $\frac{5}{6}$ $\frac{1}{2}$ $\frac{3}{4}$

$\frac{3}{8}$ $\frac{3}{4}$ $\frac{7}{10}$ $\frac{5}{6}$

17. Subtract:

$8\frac{7}{9}$ $5\frac{1}{6}$ 7

$2\frac{5}{9}$ $2\frac{1}{2}$ $3\frac{2}{7}$

Unit Test

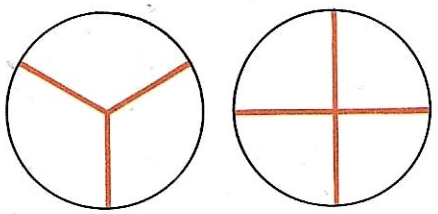
1. What is a proper fraction?

2. What is a mixed number?

3. Which of these fractions is the larger, $\frac{5}{8}$ or $\frac{5}{12}$? Tell why.

4. Which of these fractions is the smaller, $\frac{5}{4}$ or $\frac{3}{4}$? Tell why.

5. If you want like parts in the fractions $\frac{2}{3}$ and $\frac{3}{4}$, what must you do to the drawing shown here to get them?



6. Why do you call what you did in Ex. 5 by the term, finding a common denominator?

7. If you want the common denominator of $\frac{2}{5}$ and $\frac{3}{4}$, can you find it by multiplying 5 by 4? Will it be the L. C. D.? Why?

8. Add:

$\frac{5}{6} + \frac{2}{6}$ $\quad \frac{3}{4} + \frac{1}{2}$

9. Subtract:

$2\frac{3}{8} - 1\frac{1}{8}$ $\quad 3\frac{7}{9} - 2\frac{1}{3}$

10. Multiply:

$6 \times \frac{2}{5}$ $\quad \frac{2}{3} \times 13$ $\quad 4 \times 2\frac{3}{8}$

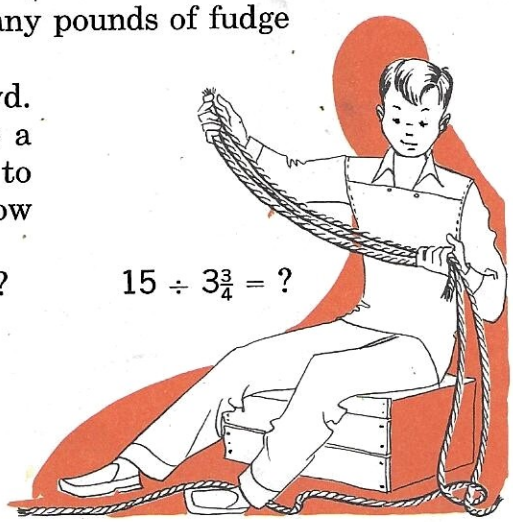
11. Divide:

$7 \div \frac{1}{2}$ $\quad 5 \div \frac{5}{6}$

12. Harry said that he had $2\frac{1}{4}$ doz. eggs on hand. If he sold $\frac{2}{3}$ of them, how many dozen did he sell?

13. Susan and 7 of her friends made $1\frac{3}{4}$ lb. of fudge each for the hospital benefit sale. How many pounds of fudge did the 8 girls make altogether?

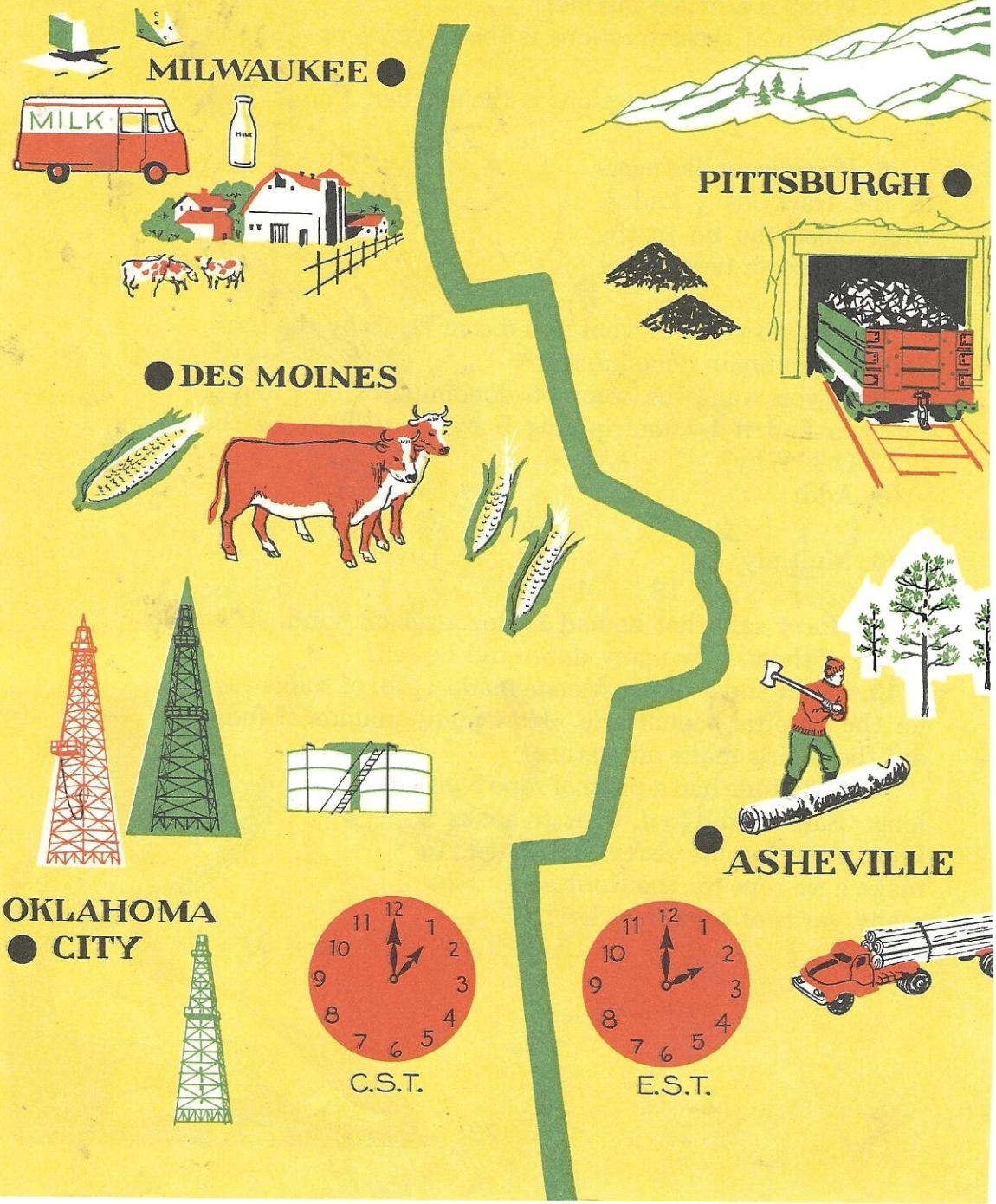
14. Mr. Stone had a piece of rope 5 yd. long. Sam used $1\frac{3}{4}$ yd. of it to make a jump rope for his sister and $2\frac{1}{3}$ yd. to make a tie rope for the front gate. How many yards of rope were left?



15. $\frac{2}{3} + \frac{1}{8} + \frac{3}{4} = ?$ $\quad 12 - 2\frac{5}{8} = ?$ $\quad 15 \div 3\frac{3}{4} = ?$

UNIT 3

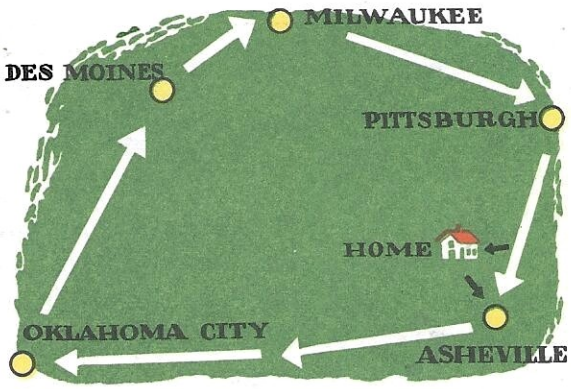
Do You Like to Travel?



Suppose you take an imaginary trip to several places in your country. Can you think of the entire distance as a whole?

Since you know that all wholes have parts, you can then think about the parts which make up this whole.

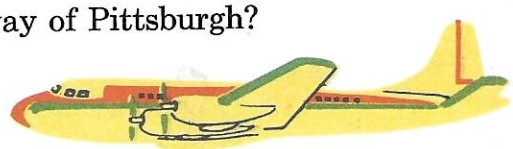
In your imaginary trip, if you know all of the parts (distances between places) and want to find the total distance, you add. Why?



On the other hand, if you know that a total distance has 2 parts and you have one of them, you would subtract the known part from the known total to get the other part. Why?

How many separate parts would you add to find the distance traveled if you follow the arrow in the sketch shown above (from Asheville, to all the cities, and back to Asheville)?

If you flew from Asheville and landed in Milwaukee, how could you use subtraction to find the remaining distance to reach Asheville again by way of Pittsburgh?



How many passengers do you think can ride in this plane? To find the total number of people aboard you can count them by ones, by twos, or perhaps add combinations of numbers. Is it necessary to add numbers, or can you always count to find how many?

1. How would you like to count in order to find these sums? Try it:

11	10	14
17	15	12

2. Add:

121 mi.	437 mi.	755 mi.	464 mi.	614	1296
243 mi.	351 mi.	132 mi.	525 mi.	372	248

Travelers Need to Use Numbers

It is 1,283 miles by highway from Oklahoma City, Oklahoma, to Raleigh, North Carolina. Look at this number to see if you understand it.

Can you read the number 1,283? You should say one thousand, two hundred eighty-three.

1. 1,283 is a four-place number. Why?
2. What place does the 3 hold?
3. What place does the 8 hold?
4. If you say eighty-three, what is the unit?
5. Do 8 tens and 3 ones give 83 ones? Why?
6. How do you change tens into ones?
7. Why do we say that the 2 is in hundreds place?
8. How do you change hundreds into tens?
9. How do you change tens into hundreds?
10. How many ones in 2 hundreds?
11. Why is 2 hundreds, 8 tens, and 3 ones equal to 283 ones?
12. Do you see 28 tens in 1,283? Where?
13. How many hundreds make a thousand?
14. Does the 1 in 1,283 hold thousands place? Why?
15. How many hundreds in 1,283?
16. Does every four-place number have a figure in thousands place?
17. Use your ruler to find the approximate scale of miles in the sketch of the road map shown on this page.
18. By means of your scale, find the approximate distances between the cities from Raleigh to Oklahoma City. Add these values to check the total of 1,283 miles.

Understanding Place Value

The whole numbers which you have studied in your arithmetic up to this point belong entirely in a system of tens. That is, each unit of a figure holding tens place is equal to 10 ones, each unit of a figure holding hundreds place is equal to 10 tens or 10×10 ones (100 ones), and so on.

millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	ones
1	6	9	2	3	7	5

1. Name the places held by the figures in the diagram.

2. How many ones in 1 thousand?

3. In a whole number, the value of one unit is how many times as large as the unit on its right?

4. In a whole number, the value of one unit is what part of the unit on its left?

5. The thousands unit is ? times the hundreds unit.

6. The thousands unit is ? times the tens unit.

7. The hundreds unit is ? times the tens unit.

8. The hundreds unit is ? times the ones unit.

9. The tens unit is ? times the ones unit.

10. The ones unit is ? of a tens unit.

11. The tens unit is ? of a hundreds unit.

12. The hundreds unit is ? of a thousands unit.

13. The ones unit is ? of a hundreds unit.

14. The ones unit is ? of a thousands unit.

15. In the number 3,452, the three is in thousands place and has 10 times the value it would have it if were in the hundreds place. Why?

16. Why is 1,000 equal to $10 \times 10 \times 10$ ones?

17. Tell why, by the tens relationship in our number system, a million is equal to one thousand thousands.

18. What does the number name ten-thousand tell you about the relation between thousands place and the place next on the left of thousands?

Reading and Writing Large Numbers

Do you know why commas are placed between groups of figures, as in 1,364,285? This is something that was invented to help you to read large numbers.

It is perfectly correct to read a number by naming all the places, as in 42,429. If you do this, you say, four ten-thousands, two thousands, four hundreds, two tens and nine ones. Using the grouping, it is forty-two thousands, four hundred twenty-nine. The groupings save words, but you must know what they mean.

This is how to group the large number 346,278,456:

millions	thousands	ones
346	278	456

You would read it, three hundred forty-six millions, two hundred seventy-eight thousands, four hundred fifty-six ones. (*Notice that you never use the word* and *as you go from one group to the next in the reading.*)

1. Read the following:
   - **a.** The city of Guacomo is 8,954 feet above sea level.
   - **b.** There are 7,058 books located in our library.
   - **c.** John lives at the corner of Fifth and Main Streets.
   - **d.** A farmer traveled 2,378 miles by auto in one month.
   - **e.** Miss Brown has had 1,873 pupils in her classes during the 50 years she has taught.
   - **f.** The number of miles traveled last year by one air pilot was 248,640.
2. Name the groups in 5,342,456.
3. Write the following:
   - **a.** Three thousand four hundred twenty-eight.
   - **b.** Fourteen thousand forty-one.
   - **c.** One hundred nine thousand one hundred nineteen.

Things to Consider When Planning a Trip

Suppose you are completing your plans to visit the areas shown on the first page of the unit.

The airplanes used for passenger transportation today are usually big, fast, and safe. The advantage found in this means of travel is the amount of time saved in going from one place to another. This is a convenience for many people. In an emergency, the traveling can be done at night, leaving daylight for useful purposes like business appointments.

Mileage, time, amount of money, and other things all have to be figured.

1. Can you estimate the distance from your home via the places on Page 74 and back to your home? Would it be 500, 1,000, 2,000, or maybe 5,000 miles?

2. Do you know what time zones are? When it is 10:00 A.M. in New York City, what is the time in Chicago?

3. What is the farthest distance that you have ever been from your home? (Give the place and about how many miles it was from your home.)

4. About how fast do commercial planes travel? Have you ever traveled 300 miles an hour?

5. At 285 miles an hour, about how far would you go in 1 minute? How far in 5 hours?

6. About how much time is saved by traveling 275 miles in 1 hour by plane instead of traveling the same distance by car at 40 miles an hour?

Comparing Travel Costs

If you are going to estimate travel costs, you will examine tables like these:

FROM PHILADELPHIA TO CHICAGO

Travel Costs—First-Class Rail vs. Air Lines

Expenditures	One Way		Round Trip	
	Standard Trains	Air Lines	Standard Trains	Air Lines
1st Class Fare	$36.84	$41.55	$70.00	$79.00
Lower Berth	8.30		16.60	
10% Tax	4.51	4.16	8.66	7.90
Breakfast $1.00 Lunch 1.50 Dinner 2.00	2.00		4.00(2)	
Tips Meal(s) Porter	.20 .75		.40 1.50	
Ground Transportation	1.50	2.35	3.00	4.70

ELAPSED TIMES: AIR, 3 hrs. 10 min. RAIL, 14 hrs. 39 min.

AIR TOURIST FARES

NEW YORK—LOS ANGELES	$99.00
BOSTON—DENVER	$81.00
DETROIT—LOS ANGELES	$86.00
LOS ANGELES—SEATTLE	$43.70
SAN FRANCISCO—HONOLULU	$125.00
OAKLAND—PORTLAND	$26.15

Transportation Tax additional.

Obtain all of the required information for one person going on a trip, starting at Asheville, North Carolina. From Asheville this person will travel to Oklahoma City, Oklahoma, then to Des Moines, Iowa, Milwaukee, Wisconsin, Pittsburgh, Pennsylvania, and return to Asheville. (Note that there are no round trips.)

Compare costs of travel by bus, train and plane. Have a committee collect the information on each method of travel and report to the class.

1. From the table given above (comparing costs by rail and air travel between Philadelphia and Chicago), find the difference between:

a. 1st class fares, one way.

b. 1st class fares, round trip.

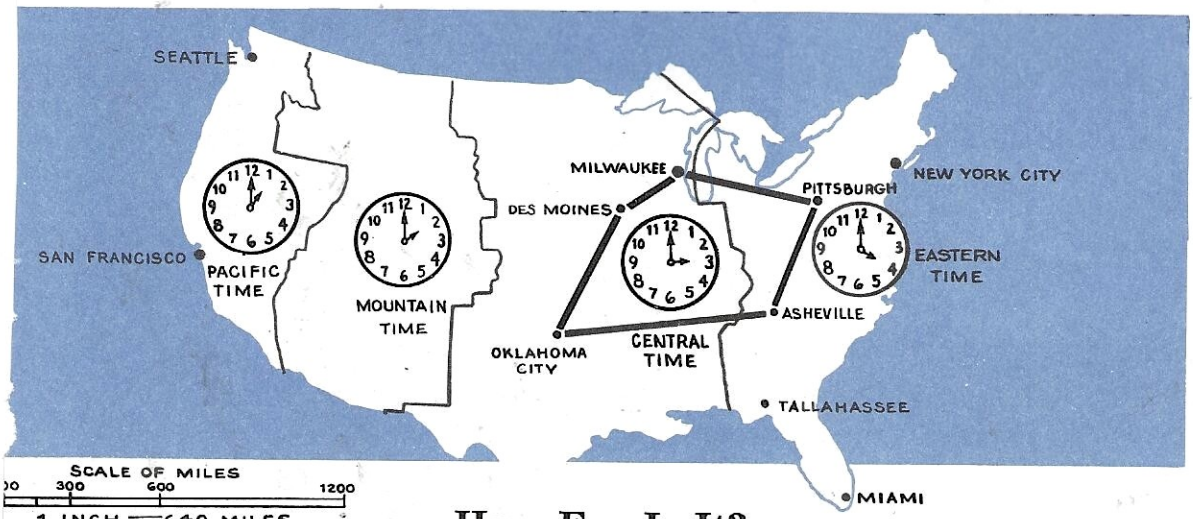
c. total costs as listed for one way.

d. total costs as listed for round trip.

2. If you travel air tourist, how much more does it cost to go to Los Angeles from New York than from Detroit?

3. What would be the total amount of fare for a family of 3 adults, traveling 1st class from Philadelphia to Chicago by plane?

4. What would it cost, traveling air tourist, to go from New York to Seattle by way of Los Angeles?



How Far Is It?

1. Use the scale of miles to find the approximate distances between these cities:

- **a.** Asheville and Milwaukee.
- **b.** Pittsburgh and Oklahoma City.
- **c.** Find the total round trip distance, when traveling the following route: Pittsburgh, Asheville, Oklahoma City, Des Moines, Milwaukee, and Pittsburgh.

2. The airline distance across the country from ocean to ocean is often stated in round numbers as 3,000 miles. Check this with the scale of miles on the map.

3. How much farther is it from Miami to Seattle than from New York to San Francisco?

4. How many hours would you allow for change of time on a trip from Tallahassee, Florida, to Albuquerque, New Mexico? Would you add or subtract this amount?

5. Draw a line 1 in. long to represent a distance of 500 mi. With this line as a scale unit, make a drawing to represent a distance of 1,500 mi. a distance of 1,750 mi.

6. If your scale is 1 in. = 60 mi., what distance would be represented by a line $3\frac{1}{2}$ in. long? by a line 12 in. long? by a line $2\frac{1}{4}$ in. long?

Time Zones

Notice the clock faces shown across the map on the previous page. Do you understand what they mean?

If you look at the map again, you will see some solid heavy lines running up and down. These are called time lines. They divide the time zones.

1. What are the names of the time zones?

2. Why must there be a difference in time for different parts of the country?

3. When it is 4 o'clock in New York City, what is the time in San Francisco?

4. If you were traveling from Philadelphia to Denver, how many times would you change your watch to show the correct time at all of the different places on the way?

5. Use your ruler and the scale of miles on the map to see about how wide a time zone is. (The location of time lines has been set by the people in the particular states for their convenience. Note the solid lines.)

6. About how fast must an airplane travel across the country so that its speed is great enough to carry the plane across each time zone in one hour?

Practice Exercises

7. Add the following:

a.	**b.**	**c.**	**d.**	**e.**	**f.**	**g.**	**h.**
322	413	530	324	102	300	443	632
456	365	407	442	303	263	306	224
311	200	362	231	694	736	550	341

8. Subtract the following:

a.	**b.**	**c.**	**d.**	**e.**	**f.**	**g.**	**h.**
465	591	222	346	894	765	937	479
321	380	105	235	634	432	836	378

Understanding Time

When you travel, you should be able to use time tables such as the one shown here.

1. What time does train 628 leave Jackson?

2. How many miles does the train travel between Detroit and New York?

Your everyday activities also require that you understand time, especially when adding or subtracting. To do this work, you need to know this table of measures.

Grand Rapids—Detroit—Buffalo—New York—Boston

Table No. 31

	628 Daily	Miles		625 Daily
	PM			AM
	1 00	0.0	Lv **Grand Rapids** (E.T.) Ar	10 40
BEELINER	y1 17	15.2	Lv Caledonia Lv	
	1 24	21.3	Lv Middleville Lv	10 13
	1 38	32.5	Lv **Hastings** Lv	9 58
	1 52	44.7	Lv Nashville Lv	9 41
	†1 56	48.3	Lv Vermontville Lv	†9 36
	2 10	59.5	Lv Charlotte Lv	9 23
	2 24	70.3	Lv Eaton Rapids Lv	9 08
	2 55	94.5	Ar **Jackson** Lv	8 40
	44			17
	3 10	94.5	Lv **Jackson** Ar	8 27
	3 55	132.7	Lv Ann Arbor Lv	7 46
	4 08	140.5	Lv Ypsilanti Lv	
	4 50	168.6	Ar **Detroit** Lv	7 05
	9 55	420.3	Ar **Buffalo** (Cent. Term.) Lv	2 20
	7 10	855.7	Ar **New York** (G. C. T.) Lv	6 00
	9 00	913.5	Ar **Boston** (So. Sta.) Lv	2 30
	AM			PM

Time Measure

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 year (yr.)
366 days	= 1 leap year

Bill timed himself to see how long it took him to do each of two different jobs. His times are shown here. How much time did he spend in all? When adding the seconds, he recorded 16 and held 1 for the minutes column. Why?

hr.	min.	sec.
2	36	40
+ 3	28	36

His sum for the minutes column was $64 + 1 = 65$.
What number should he record in this column?

3. Explain what Bill was doing here:

hr.	min.	sec.
6	24	32
− 3	15	48

=

hr.	min.	sec.
6	23	92
− 3	15	48

= ?

4. Do what the signs say:

6 hr. 21 min. 33 sec.	4 hr. 38 min. 47 sec.	1 hr. 5 min.
+ 2 hr. 38 min. 47 sec.	− 3 hr. 29 min. 53 sec.	+ 3 hr. 47 sec.

Adding and Subtracting Time Values

Learning About Daylight Saving Time

You have probably heard of Standard Time and Daylight Saving Time. Standard Time is common in the winter, Daylight Saving Time is used in the summer, and gives you an extra hour of daylight between noon and sunset. To change from Standard Time to Daylight Saving Time, you set your clocks ahead one hour.

Standard Time

Daylight Saving Time

To change from Daylight Saving Time to Standard Time you set your clocks back one hour.

If it is usually dark where you live at 7:30 P.M. by Standard Time, it will not be dark until 8:30 P.M. by Daylight Saving Time.

Setting the clock ahead 1 hr. means that the sun will rise an hour later in the morning.

1. If it is Daylight Saving Time in New York City and Central Standard Time in Chicago, what is the difference in the clock times of the two cities?

2. A train is scheduled to leave Chicago at 4:10 P.M. Central Standard Time, as shown by the station clock. All other clocks in Chicago show Daylight Saving Time. What time would this train leave by a clock outside the station?

3. If the above train took 4 hours to get to Oshkosh, Wisconsin, what time would the train arrive if Oshkosh is on Central Standard Time?

4. How much would you allow for change of time on a trip from Columbus, Ohio, to Indianapolis, Indiana, if Columbus were on Standard Time and Indianapolis were on Daylight Saving Time?

Do You Remember Addition Facts?

Some children know a great number of addition facts at sight. They remember all of the teens facts and can give the answers to such combinations as 18 + 6, 27 + 9, and 34 + 8 right off.

> The sum of two numbers that is a ones value is called a **ones fact.** The sum of two numbers that is a teens value is called a **teens fact.** Any sum of two numbers is an addition fact, but ones and teens facts are called **basic facts.**

How good are you in addition?

```
  18
  59
  67
+ 48
   ?
```

8 + 9 = 17. Do you know this fact?

17 + 7 = 24. Do you know this fact? The 17 + 7 is a hidden combination. Why?

If you don't know the fact 17 + 7 = 24, build the tens.

You can add 17 + 7 by using an ending of 4 (7 + 7 ends in 4) and bridge from the teens into the 20's. How?

```
  18
  59
  67
+ 48
   2
```

The next combination in the ones column is 24 + 8. How do you find this sum by bridging and endings?

Tell how you add the values in the tens column.

1. Tell how you add by building tens in 36 + 7 = ?

2. Add by building tens:

27	39	6	9	28	36	15	14	9
5	4	58	17	6	9	8	7	3

3. How do you add 16 + 5 using bridging and endings?

4. Add by using bridging and endings:

54	73	6	8	29	6	15	35	7
8	9	47	38	7	48	8	8	39

5. Write 27, 36, 59, and 75 in a column, and tell how you find the sums for both the seen and hidden combinations.

Building Tens in Addition

When adding some numbers in ones place, you may find an answer that contains 1 unit in the next higher place. What is this value called?

If a sum is over 9 in any place, you always have a value to add to the next higher place. Why?

1. If John travels 43 miles by bus and 19 miles by car, how far has he traveled altogether?

$$\begin{array}{r} 43 \\ 19 \\ \hline 62 \end{array}$$

$3 + 9 = 3 + 7 + 2 = 10 + 2 =$ 1 ten and 2 ones. Write the 2 ones in the ones place in the answer, holding the 1 ten.

Adding the tens: 4 tens + 1 ten + 1 ten (held) = 6 tens. Write the 6 tens in tens place in the answer. John traveled 62 miles.

2. If a plane flies 158 mi. to the first stop and 375 mi. to the next one, how far has it flown in all?

$$\begin{array}{r} 158 \\ 375 \\ \hline 533 \end{array}$$

$8 + 5 = 8 + 2 + 3 =$ 1 ten and 3 ones. Why? Write 3 in ones place in the answer, holding the 1 ten.

Now you add the tens: $5 + 7 + 1$ (held) = 13 tens = 1 hundred and 3 tens. Write 3 in tens place in the answer, and hold the 1 hundred. There will be 5 hundreds. Why?

The answer then is 533 miles.

3. Four different flights by a freight carrier were 1,268 mi., 1,135 mi., 1,046 mi., and 1,428 mi. What was the total distance covered?

$$\begin{array}{r} 1268 \\ 1135 \\ 1046 \\ 1428 \\ \hline 7 \end{array}$$

ones: $8 + 5 = 10 + 3 = 13$

$13 + 6 = 19$

$19 + 8 = 20 + 7 = 27$

Record the 7 ones, and hold the 2 tens.

1268	tens:	$6 + 3 = 9$
1135		$9 + 4 = 10 + 3 = 13$
1046		$13 + 2 = 15$
1428		$15 + 2$ (held) $= 17$
77		Record the 7 tens, and hold the 1 hundred. Why?

1268	hundreds:	$2 + 1 = 3$
1135		$3 + 0 = 3$
1046		$3 + 4 = 7$
1428		$7 + 1$ (held) $= 8$
877		Record the 8 hundreds.

1268	thousands:	$1 + 1 = 2$
1155		$2 + 1 = 3$
1046		$3 + 1 = 4$
1428		Record the 4 thousands.
4877		

1. Art's father owns several acres of land. He planted 673 seedlings for Christmas trees one year. He planted 750 the second year, 459 the third year, and 583 the fourth year. How many seedlings did he plant altogether?

2. On the tourist trip a plane flew from New Orleans to Cap Haitien, a distance of 1128 miles. From there it flew to Vera Cruz, a distance of 1396 miles. It went to Colón next, a distance of 1420 miles, and then back to New Orleans, a distance of 1343 miles. How many miles did the plane travel on this particular trip?

3. Add the following:

a.	**b.**	**c.**	**d.**	**e.**	**f.**	**g.**
1476	1314	3756	1763	1673	2878	734
1253	90	1425	232	1815	2543	1975
1241	2753	800	102	196	267	268
37	715	124	570	1357	2453	1507
2782	3450	2807	7	1846	75	1463

Practice in Addition

1. Add the following numbers by building tens:

4674	6580	7545	3810
3303	2304	1313	4105
2018	1017	1029	1076

How do you check addition? Why do you check?

2. Check your answers to the above exercises.

3. Joe and his father were both raising fields of potatoes. His father harvested 2,399 bushels of potatoes from his field, and Joe harvested 2,487 bushels from his field. How many bushels did Joe and his father harvest altogether?

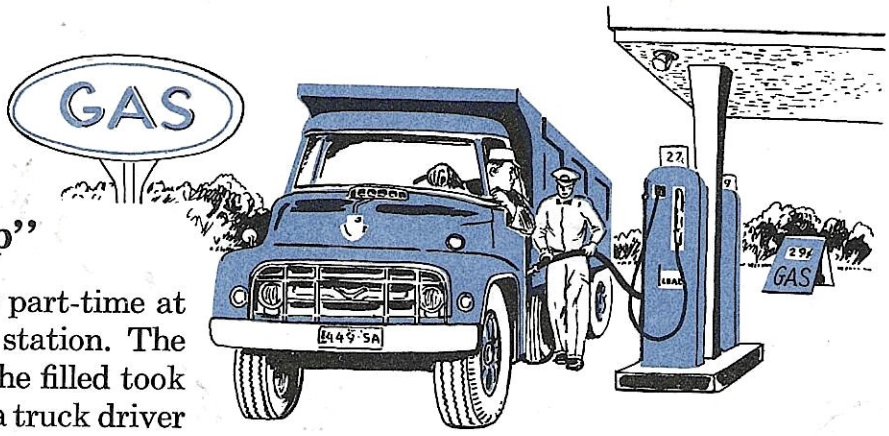
4. Last month 1,987 people used the local bus terminal. This month they expect 1,500 more to use it. How many people are expected to use the depot this month? If 901 more persons use the depot next month, how many will this make for the 3 mo.?

5. Tom's father bought 152 gallons of gasoline to use in his mower in 1954, 168 gallons in 1955, 149 gallons in 1956, and 206 gallons in 1957. What was the smallest amount of gasoline bought in any one of the 4 years? What was the total for the 4 years?

Find the sum in each of the following:

6. a.	b.	c.	d.	e.	f.	g.	h.
267	347	436	368	436	537	4356	5682
434	21	791	543	790	249	3708	936
655	745	839	672	48	851	461	2024

7. a.	b.	c.	d.	e.	f.	g.	h.
537	3768	1706	650	2759	632	672	4621
2663	2497	3472	4227	3623	304	1241	86
5461	886	287	2093	97	96	573	56
			456	386	4521	57	368



"Fill It Up"

Tom works part-time at a local service station. The first gas tank he filled took 19 gal. Then a truck driver bought 25 gal. A third customer bought 17 gal.

How many gallons did Tom sell altogether to the 3 customers?

Let's do this addition by bridging and endings.

You have learned two ways of doing this. You can call them the *building-tens-numbers* way and the *recognition* way.

To illustrate, you have:

Building tens way:

$$\begin{array}{r} 27 \\ +\ 8 \\ \hline ? \end{array}$$

27 + 3 + 5 = 30 + the ones number that is always left over (called the ending) = 30 + 5 = 35

Recognition way:

$$\begin{array}{r} 27 \\ +\ 8 \\ \hline ? \end{array}$$

You see that you have enough to bridge from a 20 to a 30 number. 7 + 8 ends in 5, so the sum is 35.

Now use both of these ways to solve Tom's problem:

$$\begin{array}{r} 19 \\ 25 \\ +\ 17 \\ \hline ? \end{array}$$

Building tens numbers:
9 + 5 = 10 + ending of ?
? + 7 = 20 + ending of ?
Finish the addition.

$$\begin{array}{r} 19 \\ 25 \\ +\ 17 \\ \hline ? \end{array}$$

Recognition:
9 + 5 bridges ? and with the ending of 4 = ?
? + 7 bridges ? and with an ending of ? = ?
Finish the addition.

1. Add, using bridging and endings two ways:

27	8	15	46	29	34	16	45
6	49	38	9	49	27	26	29

Using Bridging and Endings in Addition

If Tom's sales of gas for the 5 hours he worked Saturday morning were this set of 5 **addends**, how many gallons did he sell? (An addend is one of the numbers in an addition exercise.)

78
35
46
83
52

1. Adding by seeing bridgings and endings, you have:

78
35
46
83
52
4

Ones column, adding up: $2 + 3 = 5$; $5 + 6$ ends in 1 and bridges 1 ten, $5 + 6 = 11$; $11 + 5$, no bridging, but ends in 6, $11 + 5 = 16$; $16 + 8$ bridges 2 tens and ends in 4, $16 + 8 = 24$; record the 4 in ones place in the answer and hold the 2 tens. Why?

78
35
46
83
52
294

Tens column: $5 + 8$ bridges 1 ten and ends in 3, $5 + 8 = 13$; $13 + 4$ needs no bridging, but ends in 7, $13 + 4 = 17$; $17 + 3$ bridges 2 tens and ends in 0, $17 + 3 = 20$; $20 + 7$ needs no bridging, but ends in 7, $20 + 7 = 27$; $27 + 2$ (held) needs no bridging, but ends in 9, $27 + 2 = 29$; record the 9 in the tens place and the 2 in hundreds place. Why?

The sum is 294; Tom sold 294 gallons of gas.

2. Add and read the sums (use both ways of bridging and endings):

425	1270	268	294	1000	248
684	699	2565	678	269	1327
377	2428	3009	4724	84	450
565	476	26	2937	817	1006

3. Add (check by adding in the opposite direction):

804	$468.56	$1265.37	9	87¢	$1.56
793	29.73	688.28	422	46¢	.98
2788	246.38	49.36	6666	32¢	.09
655	40.79	158.24	755	50¢	7.28
493	123.30	1001.66	907	79¢	4.29

Adding Large Numbers

1. Do you know the names of the places in numbers larger than thousands? Begin with ones, and name the places in order as high as you can go.

Add these large numbers, then read the sums:

2.	**a.**	**b.**	**c.**	**d.**	**e.**
	678	2784	4275	6730	4444
	1565	966	3009	989	5555
	9037	8030	7777	46	6666

3.					
	15285	66248	8760	12345	1000
	6763	8973	47295	67890	10000
	87299	40837	83009	54321	90000
	984	17256	666	9867	100000

4.					
	$127.32	$2480.73	$1250.00	$1123.55	$600.99
	678.49	984.60	468.09	2374.27	7444.88
	256.38	88.57	2000.74	3046.42	868.54
	1429.40	147.66	756.49	1234.56	99.52

5. Dan's father is a salesman. He travels entirely by automobile. Dan wanted to know how many miles he had driven during the first 6 mo. of the year.

If the figures were 1,086 mi., 987 mi., 1167 mi., 689 mi., 1,272 mi., and 849 mi., what would Dan's total of his father's mileage show?

6. Write and add these values:

a. seventeen hundreds and ninety-six ones.

b. eight hundred fifty-two ones.

c. ten thousands.

d. five thousands and 28 ones.

7. Add two hundred fifty-six thousand, seven hundred nine to thirty-seven thousand, four hundred twenty.

How Do You Use Large Numbers?

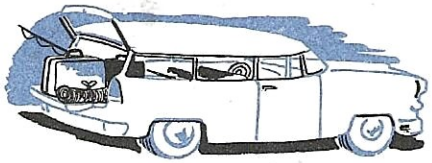
Sometimes **rounded numbers** or approximations can be used in place of accurate values. For example, if you were flying from New York City to London, would you need to think of this distance as 3,452 miles? What would be the unit of distance (50 mi., 100 mi., 1,000 mi.) which would express a sufficiently accurate value?

1. If you give the number 3,452 a rounded value to the nearest tens number, it will be either 3,450 or 3,460. To which number is it nearer?

2. You buy a railroad ticket that costs $32.86. What is the fare in round numbers to the nearest dollar?

3. What unit in round numbers are you using when you say that 2,658 mi. is about 3,000 mi.? about 2,700 mi.?

4. If you had traveled exactly 648 mi. and then had stated the distance as approximately 650 mi., why are you giving a rounded value that is accurate only to the nearest 10 mi.?



5. If you were planning a long trip by auto, would you think of the distance in terms of round numbers? Which might you use, the nearest 10 mi. or the nearest 100 mi.? Why?

6. What unit of measure are you using when you write 1,750 mi. as 2,000 mi. in round numbers?

7. The one-way fare from Philadelphia to Chicago (1st class) is $36.84. What is this value when rounded to the nearest 10¢?

8. Find the distance on a road map from your home to Chicago. Write the number that tells the miles correct to the nearest ten; to the nearest hundred.

Rounding Numbers

1. What would you write if you rounded 78 mi. to the nearest 10 mi.? Why?

When rounding a number to the nearest ten, you consider the figure in ones place. If it is 5 or more, you make the figure in tens place 1 larger. If it is less than 5, you write a zero in ones place.

2. Round the following numbers to the nearest ten:

28 65 96 71 254 138 429 503

3. To which round number in hundreds is 426 mi. nearest?

When rounding a number to the nearest hundred, you consider the figures in the tens and ones places. If it is fifty or more, you make the figure in the hundreds place 1 larger. If it is less than 50, you write zeros in the tens place as well as in the ones place.

4. Round the following numbers to the nearest hundred:

312 976 255 73 649 463 850 501

5. It is often stated that the distance around the earth at the equator is 25,000 mi. Do you think the actual value has been rounded? Check it by finding the distance given in your school encyclopedia. Tell whether the 25,000 is rounded to the nearest hundred or thousand.

6. Round the following numbers to the nearest hundred, than add them:

325	510	615	363	640	910	453
475	360	141	444	570	670	607
681	290	585	793	790	220	849

7. Add the numbers in Exercise 6 as they stand, then round the answers to the nearest hundred. How do the answers compare with those you first found?

A Test in Addition

1. Here is Tom's banking schedule for a five-week period. The first week he put $32.00 into his account. The following two weeks he deposited $20.00 each week. The next week he deposited $15.00, and the last week $17.00. How much money did Tom deposit during the 5 weeks?

2. Tom wished to earn $57.00 to buy a bicycle. He earned $7.00 one week, $12.00 the next week, $14.00 another week, and $21.00 the fourth week. Has he earned enough money to buy the bicycle?

3. Tom read the speedometer of an automobile parked at the service station where he works. It read 20,463 miles. The driver had driven 468 miles that day and 398 miles the preceding day. How far had he traveled in the two days?

4. Add, checking each exercise by adding in the opposite direction:

a.	**b.**	**c.**	**d.**	**e.**	**f.**
2368	4783	5678	3333	16296	96278
3564	9675	9876	4444	4278	4827
6789	4270	4004	8888	14599	93989

g.	**h.**	**i.**	**j.**	**k.**	**l.**
$465.75	$25.62	$6.00	$.75	$40.00	67¢
328.42	985.39	8.99	.97	1.59	49¢
97.89	49.98	17.88	.46	100.32	88¢

5. What is the name of the coin that has the value of the place of the 7 in $8.75? Why?

6. Suppose a traveling salesman has a record of the following distances which he traveled in the last year:

Air	26,278 mi.	Car	15,689 mi.
Train . . .	2,785 mi.	Boat	1,948 mi.
Bus	1,296 mi.	Taxi	893 mi.

How far did he travel in the one year?

How Much Farther to Go?

You have already learned how to subtract when using three-place numbers. The same method is used when subtracting larger numbers.

Suppose you are flying this distance and know that you have already covered 1,460 miles. What is the remaining distance?

```
  3680
- 1460
  2220
```

You subtract the ones from ones, tens from tens, and so on.

1. Let's examine this exercise:

```
 3456 mi. — minuend
- 2678 mi. — subtrahend
    ?  mi. — difference or remainder
```

a. How do you get 8 in ones place in the remainder?

b. What change did you make in the 5 tens?

c. How must you change the 4 hundreds in order to have enough tens in the minuend (top number) so that you can subtract the 7 tens in the subtrahend (the bottom number)?

d. Explain what you must do to the thousands place in the minuend in order to subtract the hundreds.

2. What is the remainder in this subtraction?

```
  3462 mi.
- 1584 mi.
```

4 from 12 = ?; how is the 12 obtained?
8 from 15 = ?; how is the 15 obtained?
5 from 13 = ?; how is the 13 obtained?
1 from 2 = ?; why only 2 in thousands place?
The remainder is ? miles.

3. Subtract:

2743	6832	3726	5793	7235	3942	4883	8673
1907	4786	2766	2794	5276	3683	2465	2664

How Do You Subtract?

What is your method of doing subtractions? Do you, **a**, take the values of the figures in the subtrahend (the number you are subtracting) from the values in the same places in the minuend (the number from which you are subtracting)? In this process you use subtraction facts. Or, do you, **b**, add values to the figures in the subtrahend to give ones or teens values in these same places in the minuend? In this process you use addition facts.

In either **a** or **b** you often need to regroup the values in the minuend. Why?

Look at these examples. In A, regrouping is unnecessary. In B, there must be regrouping.

A	B
687	724
− 245	− 367

Tell which of the following is your method:

Regrouping Unnecessary:

Using subtraction facts:

687	ones: 7 − 5 = 2
− 245	tens: 8 − 4 = 4
442	hundreds: 6 − 2 = 4

Using addition facts:

687	ones: 5 + ? = 7
− 245	tens: 4 + ? = 8
442	hundreds: 2 + ? = 6

Regrouping Necessary:

Using subtraction facts:

724	14 − 7 = 7
− 367	11 − 6 = 5
357	6 − 3 = 3

Using addition facts:

724	7 + ? = 14
− 367	6 + ? = 11
7	3 + ? = 6

Can you think the regrouping or do you have to use a "crutch" like the following?

$$\begin{array}{r} {\scriptstyle 6\ 11\ 14} \\ \not{7}\not{2}\not{4} \\ -\ 3\ 5\ 7 \\ \hline ?\ ?\ ? \end{array}$$

Why must there be teens values in some places in the minuend? Where is the 14 obtained? the 11? the 6?

1. Copy and subtract: 975 − 233, 856 − 140, 785 − 283, 640 − 320, 488 − 259, 654 − 263, 564 − 296, 745 − 456, 980 − 299

Zeros in Subtraction

Naming the places will help you to understand the meaning of a subtraction with zero in the minuend. In the following subtraction you need more ones in order to subtract in ones place. In other words, you must regroup.

But there are no tens in tens place from which 1 ten is usually taken to give the needed ones. Reading the places as the values are given, you have 4603 = 4 thousands, 6 hundreds, 0 tens, and 3 ones.

Then by regrouping:

4,603 = 4 thousands, 5 hundreds, 10 tens, and 3 ones. Why?
4,603 = 4 thousands, 5 hundreds, 9 tens, and 13 ones. Why?
4,603 = 3 thousands, 15 hundreds, 9 tens, and 13 ones. Why?

This last form is usable. So:

4603	=	3 thousands	and	15 hundreds	and	9 tens	and	13 ones
− 1875		1 thousand	and	8 hundreds	and	7 tens	and	5 ones
		?	and	?	and	?	and	?

The difference is 2,728. Which method did you use?

```
  4603
− 1875
  2728
```

By thinking the regrouping and taking the steps as you come to them, this is what you do:

To get more ones for ones place, 1 hundred is taken from the 6 hundreds and used as 9 tens and 10 ones. Thus, you have 13 ones.

13 − 5 = 8. Write 8 in ones place in the difference.

9 − 7 = 2. Write 2 in tens place in the difference.

You need more hundreds. Using 1 of the 4 thousands as 10 hundreds, you have 15 − 8 (remember, you used 1 of the 6 hundreds) = 7. Write 7 in hundreds place in the answer.

3 − 1 = 2. Write 2 in thousands place in the answer. The difference is 2,728.

1. What values would you have in the different places in order to subtract 2,648 from 5,005?

Practice Problems and Exercises

1. If your proposed visit to various places showed flight distances of 1,268 mi., 1,135 mi., 1,046 mi., and 1,428 mi., how much greater than the shortest was the longest distance? Find the difference in the mileage as you go from one place to the next, in the order given.

2. If we say the distance across the Atlantic Ocean is 3,452 mi. and the distance across the United States is 3,194 mi., which is the greater? How much greater?

3. If 100 gallons of petroleum will produce 44 gallons of gasoline and 36 gallons of fuel oil, how many gallons are left for other products?

4. One oil field in Texas has about 25,000 producing wells, while another in Oklahoma has about 18,700 wells. How much greater is the number of wells in the Texas field than in the Oklahoma field?

5. A good dairy cow may weigh 1,000 pounds and produce 7,500 pounds of milk in 1 year. How many more pounds of milk per year does she produce than her weight?

6. Since the introduction of the hybrid variety of corn to the U.S. corn belt, the production per acre has increased greatly. If a farmer's yield on a 40 A. field increased from 55 bu. to 112 bu. per acre, what was his total increase?

7. Subtract, using the addition facts method:

a.	b.	c.	d.	e.	f.	g.
5365	6732	1654	3542	8457	7366	5716
4278	3843	1565	1658	5868	3898	2948

Checking Subtraction

Do you remember how the minuend, the subtrahend, and the difference in a subtraction example are related? That is, if you know that $9 - 7 = 2$, then you also know that $9 - 2 = 7$ and $2 + 7$ (or $7 + 2$) $= 9$. These last two facts can be used as checks on the first.

Look at this example:

Solution:	Checks:	
365	365	167
− 198	− 167	+ 198
167	198	365

Estimating the difference by using round numbers is also a rough check. Let's round the numbers in the above example to the nearest ten:

365	370
− 198	200
167	170

Of course the rounded values are not exact. But we are off by only a value of 3. This method tells us that an answer of 267 is incorrect.

1. Subtract and check by addition:

984	756	805	640	1063	4004
459	367	248	370	578	2386

2. Subtract and check by another subtraction:

856	1057	903	1620	4058	10837
234	1049	658	1275	2169	6954

3. Use estimates as rough checks in these exercises: **a.** rounding to nearest ten, **b.** rounding to nearest hundred.

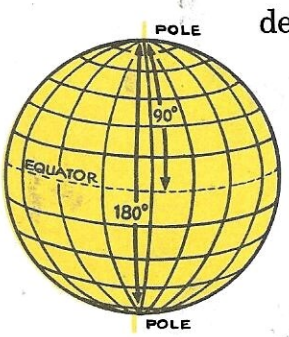
676	708	1257	1239	978	4072
− 483	− 563	− 849	− 812	− 619	− 1859

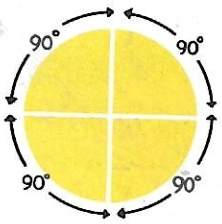
Which is the better check, a. or b.? Can you tell why?

New Three-Place Numbers

You often read in the papers about a ship or plane in distress at sea. How do they find where these craft are?

If you examine a globe, you will see some numbers with labels that may be new to you. These are used in navigation. For example, you will find 60°, 75°, and others. These little zeros at the right of the numbers indicate distances in terms of a unit on the earth's surface called a degree. There are 360 **degrees** around a circle = 360°.





The equator is considered to be a circle. Also, a line around the earth passing through the poles is considered a circle. When each of these total distances is divided into 360 equal parts, there is 1° in each part.

Degrees are divided into 60 equal parts, each part being called a **minute** (1′). Then the minutes have 60 equal parts, each being called a **second** (1″).

The earth's surface is measured east and west from a line passing through the poles and passing through Greenwich, England. These lines running north and south are called lines of *longitude*.

There are lines passing around the earth in the same direction as the equator. They are called lines of *latitude*.

1. Why can 6° 15′ 36″ be considered a three-place number?

2. Add:

$$\begin{array}{r} 8^\circ\ 32'\ 46'' \\ 5^\circ\ 42'\ 18'' \\ \hline \end{array} = \begin{array}{r} 8^\circ\ 32'\ 46'' \\ 5^\circ\ 42'\ 18'' \\ \hline 13^\circ\ 74'\ 64'' \end{array} = 14^\circ\ 15'\ 4''$$

Why?

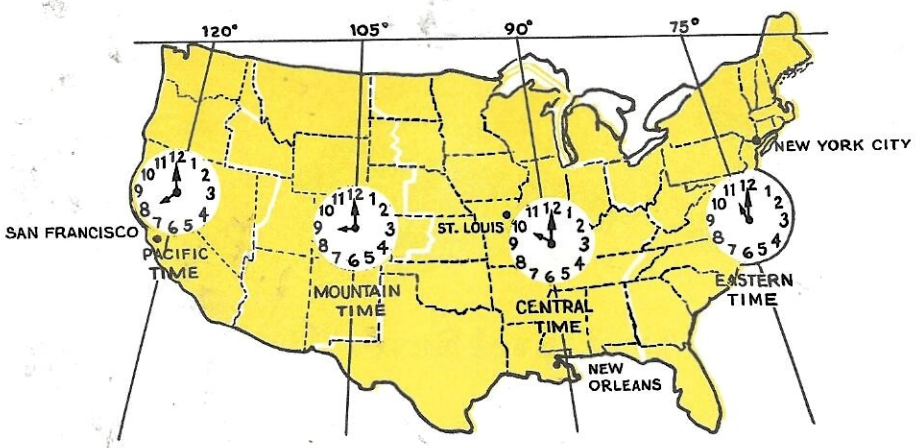
$$\begin{array}{r} 63^\circ\ 14'\ 48'' \\ 29^\circ\ 38'\ 82'' \\ \hline \end{array}$$

3. Subtract:

$$\begin{array}{r} 15^\circ\ 18'\ 32'' \\ 8^\circ\ 28'\ 54'' \\ \hline \end{array} = \begin{array}{r} 14^\circ\ 77'\ 92'' \\ 8^\circ\ 28'\ 54'' \\ \hline ?\ \ ?\ \ ? \end{array}$$

Why?

$$\begin{array}{r} 74^\circ\ 19'\ 27'' \\ 36^\circ\ 28'\ 52'' \\ \hline \end{array}$$



Time and Longitude

1 degree of longitude = 4 minutes of time
15 degrees of longitude = 1 hour of time
360 degrees of longitude = 24 hours of time

1. At New Orleans, La., one degree of longitude equals about 60 miles. Five degrees would be how many miles?

2. The difference between the 3,000 mi. that is often stated as the width of the U.S. and a value of 3,150 mi., estimated from the degrees in the sketch, is how many miles?

3. If it is 5:15 P.M. in New York City, what time is it in San Francisco?

4. What time is it in St. Louis, Missouri, when it is 11:00 A.M. in Philadelphia, Pennsylvania?

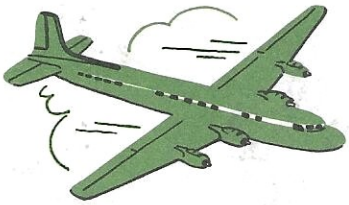
5. In which time zone do you live? About what longitude is it? About how many units of longitude is it from your home to San Francisco? to Boston?

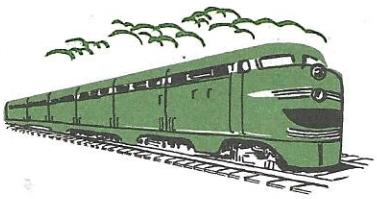
6. Add:

6 hr. 36 min. 18 sec.	7 hr. 8 min. 56 sec.
9 hr. 28 min. 15 sec.	9 hr. 53 min. 14 sec.

7. Subtract:

12 hr. 28 min. 42 sec.	16 hr. 34 min. 24 sec.
9 hr. 16 min. 28 sec.	8 hr. 42 min. 38 sec.





Air Travel Saves Time

	Airplane	Railroad	Automobile
Chicago to New York	2 hrs. 45 min.	20 hrs. 30 min.	24 hrs. 40 min.
Chicago to New Orleans	3 hrs. 25 min.	16 hrs. 15 min.	23 hrs. 20 min.
Chicago to San Francisco	6 hrs. 15 min.	39 hrs. 30 min.	53 hrs. 10 min.

1. How much time is saved if you go from Chicago to New York by air rather than by automobile?

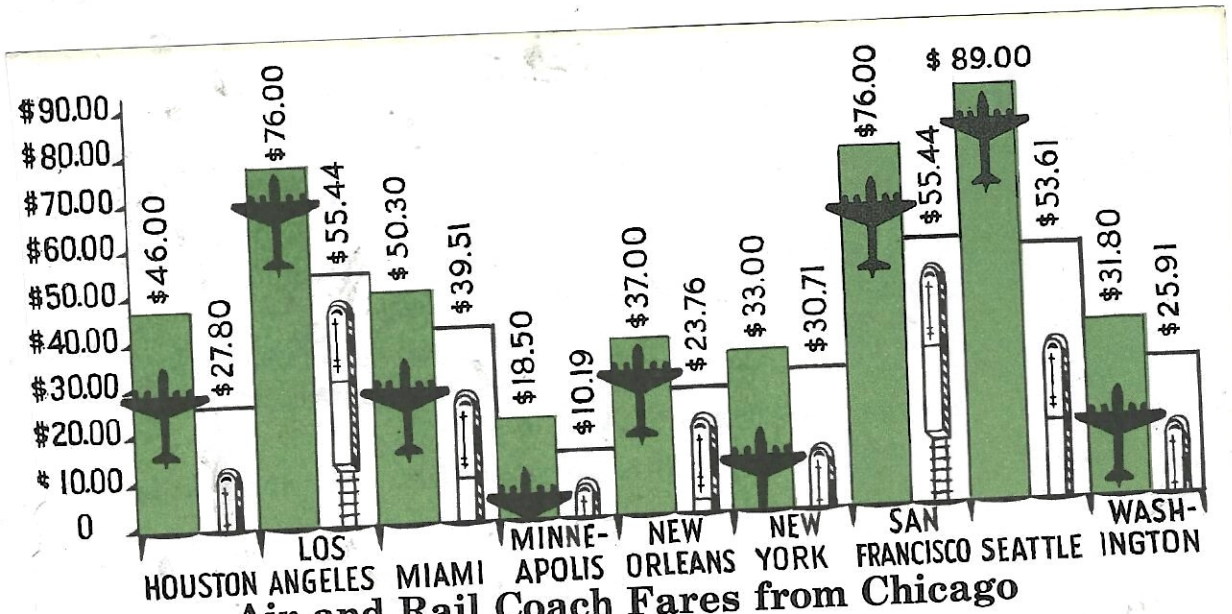
2. How much time does the airplane save over the railroad when you go from Chicago to San Francisco?

3. Which way would you prefer to travel if you were to take a long trip? Why?

4. If you had a business engagement in New Orleans in 30 hours, would you have time to drive? Would you be likely to drive? Why or why not?

5. How much time would you save if you were to take the train from Chicago to New York rather than the automobile?

You probably would want to take a plane for one trip, a car for another, or even a train for another trip. The way you travel depends on many factors. One factor is money. Another factor is time. If you are in a great hurry, you would most likely take a plane. If you were planning on seeing the local sights, you might want to take a car.



Air and Rail Coach Fares from Chicago

The bar graph is made by putting the amount of money in a scale at the left of the graph. Each amount is then filled in so that the top of the bar reaches exactly to the amount of money represented by the scale.

Can you tell at a glance which fares are the highest and which are the lowest?

How much is the fare to Seattle from Chicago by rail, including the tax?

How much does it cost to go to Seattle from Chicago by air? by rail? How much is the saving?

If you were to plan two trips from Chicago, the first to Los Angeles, and the second to Miami, would you save enough on the train fare from the first trip to pay the plane fare on the second trip? On which trip would you find the rail and the air coach fares more nearly alike? the most different?

In planning the costs of travel there are extras such as meals, tips, travel to the depot or airport, etc. How should these be figured?

1. Make a bar graph of the distances from your home to Boston, New York, Atlanta, New Orleans, San Francisco, and Alaska.

Unit Review

1. If the distance between each base in a softball game is 65 feet, what is the complete distance from home plate around the sacks to home plate again?

2. In a softball game you hit a ball out of the park for a home run. With a man on first base, how much farther do you have to run than he to score?

3. If the left field fence is 200 feet away and you hit a fly ball 112 feet, how much farther would you have to hit the ball to make a home run?

4. You kept track of your times at bat for a summer and in all you batted 184 times, getting 40 hits. How many times did you get put out that summer?

5. During the summer you made 40 hits; Tom made 52 hits; Billy made 32 hits; Dan made 35 hits; and Carl made 29 hits. Altogether how many hits did you and your friends make?

6. Mickey Mantle of the New York Yankees hit a ball 565 feet (the longest ball ever hit in the majors). If the fence was only 375 feet from home plate, how far out of the park did he hit the ball?

7. Add:

115	468
220	532

8. Subtract:

705	813
696	604

9. Add:

375 tons	533 feet	896 bushels	305 hours
468 tons	691 feet	327 bushels	460 hours

10. Subtract:

976 miles	284 days	461 barrels	493 pounds
345 miles	172 days	375 barrels	297 pounds

11. Express the following in round numbers:

to the nearest ten:

791 247 460 1109

to the nearest hundred:

5492 4960 5049 9453

Unit Test

1. There is a bug called the 17-year locust that affects farmers' crops. The bugs last appeared in the spring of 1953. They came out of the ground, shed their old skins, laid eggs, and went underground again. When will the bugs next appear above ground?

If you were 10 years old in 1953, how old will you be when the bugs come out of the ground again?

2. If the corn crop of the state of Iowa in a year is valued at 500 millions of dollars and the meat products at 225 millions, what is the total value of these two products? How much greater is the value of the corn crop than the meat products for a year?

3. Iowa produces some coal. If one part of the state mines 750,000 tons and another part a million tons a year, what is the difference in these two yields?

4. More than one-half of the cheese produced in the United States comes from the state of Wisconsin. Some recent records showed the following yearly productions in a few average-sized communities:

A. 56,000 lb.	C. 76,000 lb.	E. 37,000 lb.
B. 42,000 lb.	D. 28,000 lb.	F. 29,000 lb.

What was their total production?

5. The great eastern coal field covers about 70,000 square miles. The total area of coal fields in the U.S. and Alaska is about 330,000 square miles. How much greater is the total area than the eastern field?

6. $27° 36' 42'' + 28° 15' 28''$

7. $124° 15' 36'' - 49° 28' 42''$

8. $16475 + 13909$

9. $6007 - 2198$

10. Mr. Cook took his family on four trips. If the distances were 1,346 mi., 874 mi., 1,693 mi., and 4,028 mi., what was the total amount of traveling that was done by the Cook family?

UNIT 4

Managing Your Own Affairs

Notice the girl walking through the doorway in the picture on the opposite page. Would you say that plans had been made for a trip to some vacation spot by her family and that she is getting things ready for the trip?

See the paper boy on his bike? Do you suppose that he makes an estimate of what he earns each week or each month? How would he have to use numbers in order to make a fair estimate of his weekly earnings?

The boy across the street must be making a home for a new pet. Before he started making the fence, he had to figure where he wanted to erect it, its length, and its cost.

If you had to do the figuring, how would you figure the cost of 128 feet of fencing at 33¢ a foot? Would you decide to add 33¢ as many as 128 times? Why or why not?

Do you know how to manage your activities?

Suppose that you were planning to fence a plot of ground for a pony. You measure the distance around the plot. Then you decide that you need 3 strands of wire of the measured length. If the distance around the plot is 4,285 ft., how do you find how much wire you need?

a. What are the names of the places in 4,285?

b. When you multiply the 5 ones by 3, what do you get as places in the result?

c. If you multiply the 2 hundreds by 3 and have 2 hundreds held from the tens, what figure must you record in the answer? In what place will it be recorded?

In multiplication: the number to be multiplied (sometimes called the **multiplicand**) is the how-much of some measured quantity.
The number you are multiplying by (sometimes called the **multiplier**) is the how-many times you are using the other value.
The answer in multiplication is sometimes called the **product.**

Building a Playhouse

Sam and Sue decided to build a playhouse for the small children in the neighborhood.

What do you think they needed to know in order to build the house?

Before any building was done, Sam's father told him that they could use the lumber which was stored in his garage. They found several long boards there, all the same size.

One of the first things they wanted to know was how many inches should be cut from one end of one of the long boards to give them 3 pieces 13 in. long. They used the names of the places in 13 to find the answer.

$$\begin{array}{r} 13 \\ \times\ 3 \\ \hline \end{array} = \begin{array}{r} 1 \text{ ten} + 3 \text{ ones} \\ \times\ 3 \\ \hline 3 \text{ tens} + 9 \text{ ones} = 39 \text{ in.} \end{array}$$

$$\begin{array}{r} 13 \text{ in.} \\ \times\ 3 \\ \hline \end{array}$$

Can you do the multiplication by thinking the place values?

$$\begin{array}{r} 13 \text{ in.} \\ 3 \\ \hline 39 \text{ in.} \end{array}$$

3×3 ones $= 9$ ones, and 3×1 ten $= 3$ tens. Why are these values written as they are in the answer?

1. $\begin{array}{r} 34 \text{ in.} \\ \times\ 2 \\ \hline \end{array}$ $2 \times 4 = ?$ Where is this value placed?
   $2 \times 3 = ?$ Where is this value placed?

2. What are the place values that you must multiply in the exercise 2×24 ft.?

3. What is the highest place value that you will get when you multiply 32 ft. by 2? Why?

Checking Old Learnings

Do you have a way of figuring multiplication facts which you cannot remember?

Suppose that you did not know the fact $7 \times 9 = 63$. You could find seven 9's by separating the 9 into parts and multiplying them separately. Of course, you will have to add the results of multiplying the parts.

Let's look at seven 9's:

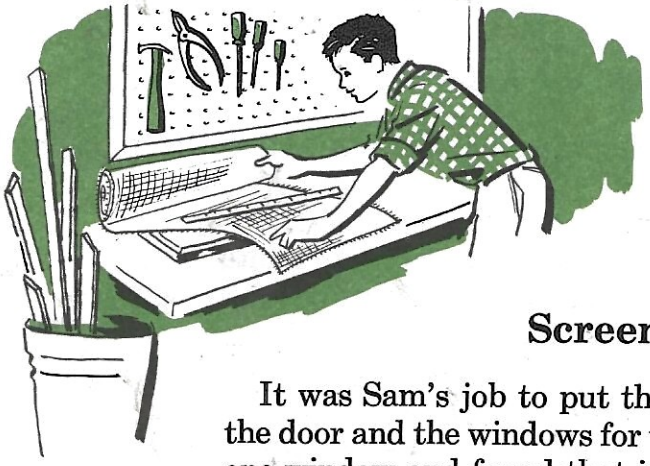
$7 \times 9 = (7 \times 4) + (7 \times 5) = 28 + 35 = 63$

Use a method like this for any of the following multiplications:

	a.	b.	c.	d.	e.	f.	g.	h.
1.	8 × 9	7 × 6	8 × 7	9 × 6	7 × 9	5 × 8	10 × 6	9 × 8
2.	9 × 7	6 × 8	8 × 5	6 × 10	7 × 8	9 × 9	10 × 8	6 × 9
3.	6 × 7	5 × 6	8 × 6	5 × 9	11 × 4	7 × 10	11 × 8	12 × 6
4.	7 × 5	11 × 5	10 × 10	12 × 5	7 × 4	8 × 4	11 × 9	10 × 7
5.	12 × 4	11 × 7	9 × 5	8 × 8	7 × 7	11 × 6	12 × 3	10 × 9
6.	8 × 3	11 × 8	10 × 6	11 × 3	9 × 4	8 × 10	5 × 10	4 × 9

Copy these exercises and multiply:

7.	$12 \times 10 = ?$	$7 \times 9 = ?$	$8 \times 7 = ?$
	$9 \times 8 = ?$	$6 \times 11 = ?$	$12 \times 7 = ?$
8.	$11 \times 9 = ?$	$10 \times 12 = ?$	$9 \times 10 = ?$
	$11 \times 11 = ?$	$8 \times 12 = ?$	$11 \times 10 = ?$
9.	$12 \times 8 = ?$	$9 \times 12 = ?$	$11 \times 8 = ?$
	$7 \times 11 = ?$	$11 \times 11 = ?$	$12 \times 12 = ?$



Screens for the Play House

It was Sam's job to put the screening on the frames of the door and the windows for the playhouse. He measured one window and found that it was 36 in. high. He would need twice this amount for the 2 windows.

How would you find how much wire screening Sam needs for the windows? Would you add, or would you multiply? By addition:

$$\begin{array}{r} 36 \\ +\ 36 \\ \hline \end{array}$$

6 ones + 6 ones = ? ones. What do you record in ones place in the answer? How many tens do you get?

By multiplication:

$$\begin{array}{r} 36 \\ \times\ 2 \\ \hline \end{array}$$

Here 36 is thought of as 3 tens and 6 ones. 2 × 6 ones = 12 ones.

Write 2 in ones place in the answer. Hold 1 ten; there will be other tens. Why? 2 × 3 tens = 6 tens. 6 tens + 1 ten (held) = 7 tens. Write 7 in tens place in the answer. 7 tens and 2 ones make 72.

Sam needs 72 inches of wire screen.

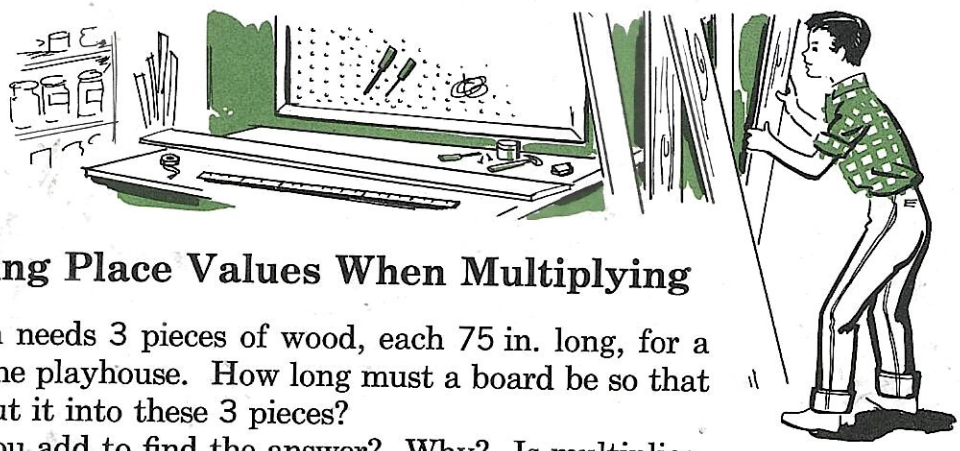
3 tens + 6 ones
× 2
? tens + ? ones

$$\begin{array}{r} 36 \\ \times\ 2 \\ \hline 72 \end{array}$$

Multiply:

1. $\begin{array}{r} 124 \\ \times\ 2 \\ \hline \end{array}$ = 1 hundred + 2 tens + 4 ones, × 2 = ? hundreds + ? tens + ? ones = ?

2. $\begin{array}{r} 226 \\ \times\ 3 \\ \hline \end{array}$ = 2 hundreds + 2 tens + 6 ones, × 3 = ? hundreds + ? tens + ? ones



Thinking Place Values When Multiplying

1. Sam needs 3 pieces of wood, each 75 in. long, for a part of the playhouse. How long must a board be so that he can cut it into these 3 pieces?

Can you add to find the answer? Why? Is multiplication a shorter way?

$$\begin{array}{r} 75 \text{ in.} \\ \times\ 3 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 75 \text{ in.} \\ \times\ 3 \\ \hline 225 \text{ in.} \end{array}$$

3×5 ones $= 15$ ones. What do you record in ones place in the product? What value is held for another place? What place?

3×7 tens $= 21$ tens. 21 tens $+ 1$ ten (held) $= 22$ tens or 2 hundreds $+ 2$ tens. Why? How are these values recorded in the product?

Multiply by thinking place values:

2.

a.	b.	c.	d.	e.	f.	g.	h.
16	12	36	15	72	36	26	58
$\times\ 2$	$\times\ 8$	$\times\ 3$	$\times\ 4$	$\times\ 4$	$\times\ 5$	$\times\ 3$	$\times\ 2$

3. Multiply 4×128:

$$\begin{array}{r} 128 \\ \times\ 4 \\ \hline \end{array} = \begin{array}{r} 1 \text{ hundred} + 2 \text{ tens} + 8 \text{ ones} \\ \times\ 4 \\ \hline 5 \text{ hundred} + 1 \text{ ten} + 2 \text{ ones} = 512 \end{array}$$

Why?

4. Multiply by thinking place values:

$$\begin{array}{r} 275 \\ \times\ 3 \\ \hline ? \end{array}$$

3×5 ones $= ?$ ones. What is recorded and what is held?

3×7 tens $= ?$ tens; $?$ tens $+ ?$ tens (held) $=$ $?$ tens. What is recorded and what is held?

3×2 hundreds $+ ?$ hundreds (held) $= ?$

5. Multiply by thinking place values:

138	280	136	235	179	226	486	209
$\times\ 4$	$\times\ 3$	$\times\ 5$	$\times\ 4$	$\times\ 3$	$\times\ 4$	$\times\ 2$	$\times\ 4$

Planting a Strawberry Patch

1. Harry and Jane decided to plant a new strawberry patch. They planned to have 15 rows across the garden. If they need a space 3 ft. wide for each row, how many feet wide will their garden be?

$$\begin{array}{r} 3 \text{ ft.} \\ \times\ 15 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times\ 15 \\ \hline 15 \end{array}$$

There are to be fifteen 3's, not three 15's. Why? Use the multiplier 15 in parts: 15 = 10 and 5. First, multiply by 5:

5 × 3 ones = 15 ones = 1 ten and 5 ones.

How are these values recorded?

$$\begin{array}{r} 3 \\ \times\ 15 \\ \hline 15 \\ 30 \\ \hline 45 \end{array}$$

This is called the **first partial product.**

10 × 3 ones = 30 ones = 3 tens and 0 ones.

How are these values recorded for the **second partial product?**

The product is 45 feet. Why?

2. How many plants would Harry and Jane need if they planted 16 rows with 8 plants in each row? (Study the following way to record values.)

$$\begin{array}{r} 8 \\ \times\ 16 \\ \hline 8 \end{array}$$

6 × 8 ones = 48 ones. Write 8 in ones place in the product. Notice that the 4 tens have not been recorded as was done in the previous example. You can hold them until you find 10 × 8 ones = 80 ones = 8 tens.

$$\begin{array}{r} 8 \\ \times\ 16 \\ \hline 128 \end{array}$$

Now you add the 8 tens and the 4 tens (held): 8 tens + 4 tens = 12 tens = 1 hundred + 2 tens.

Harry and Jane will need 128 plants.

3. Multiply (record your products two ways):

4	5	6	8	4	7	3	2	8
13	15	14	12	18	14	19	17	18

4. How many strawberry plants will Harry and Jane need altogether if they plant 15 rows with 15 plants in each row?

$$\begin{array}{r} 15 \\ \times\ 15 \\ \hline \end{array}$$

The multiplier is 15. You could use the parts 8 and 7 in the multiplication. But it is easier to see 15 as 10 and 5. Why?

$$\begin{array}{r} 15 \\ \times\ 15 \\ \hline \end{array} = \begin{array}{r} 1 \text{ ten} + 5 \text{ ones} \\ \times\ 5 \\ \hline 5 \text{ tens} + 25 \text{ ones} \\ \text{or} \\ 7 \text{ tens} + 5 \text{ ones} \end{array} + \begin{array}{r} 1 \text{ ten} + 5 \text{ ones} \\ \times\ 10 \\ \hline 10 \text{ tens} + 50 \text{ ones} \\ \text{or} \\ 15 \text{ tens} + 0 \text{ ones} \\ \text{or} \\ 1 \text{ hundred} + 5 \text{ tens} + 0 \text{ ones} \end{array}$$

Adding partial products:

$$\begin{array}{r} 7 \text{ tens} + 5 \text{ ones} \\ 1 \text{ hundred} + 5 \text{ tens} + 0 \text{ ones} \\ \hline ? \qquad ? \qquad ? \end{array} = ? \text{ plants}$$

Thinking the values in the multiplication:

$$\begin{array}{r} 15 \\ \times\ 15 \\ \hline 75 \end{array}$$

You can think: 5×5 ones $= 25$ ones. Write 5 in ones place in the first partial product, and hold the 2 tens. Why?

5×1 ten $= 5$ tens. 5 tens + 2 tens (held) = 7 tens. Write 7 in tens place in this partial product.

$$\begin{array}{r} 15 \\ \times\ 15 \\ \hline 75 \\ 150 \\ \hline 225 \end{array}$$

You can think: 10×5 ones $= 50$ ones. Write 0 in ones place in the second partial product, and hold the 5 tens. Why?

10×1 ten $= 10$ tens. 10 tens + 5 tens (held) = 15 tens = 1 hundred and 5 tens. Write 5 in tens place and 1 in hundreds place in this partial product.

Adding the partial products, you get 225.

Do you need to record the zero (in ones place) in each of the second partial products? Why or why not?

1.

4	6	7	5	13	14	16	17
$\times$ 15	$\times$ 12	$\times$ 9	$\times$ 14	$\times$ 13	$\times$ 12	$\times$ 16	$\times$ 14

Multiplication Made Easier

The meanings in multiplication help you to understand the shortcuts of multiplication. Look at the exercise 32×48:

Step 1: If you used meanings, you would have to say 2×8 ones = 16 ones, and so on. But you now know that 2×8 will first give the same kind of units as the 8. So, the 6 in 16 will go in the same place in the partial product as the 8 and can be recorded under it. You hold the 1 in 16.

Then $2 \times 4 = 8$; $8 + 1$ (held) = 9. This 9 is of the same units as the 4 and will be recorded under it in the partial product. Why?

Step 2: If you used meanings here, you would say that 30×8 ones = 240 ones. Then 0 would be recorded in ones place in the partial product, and the 24 tens would be held.

But, you can now see that there is no need of recording the 0 and, instead of thinking $30 \times 8 = 24$ tens, you will multiply 8 by the 3 in tens place ($3 \times 8 = 24$) and know that you get units which are one place higher than those of the 8, or that ones are changed to tens in meaning. So, the 4 in 24 will be recorded under the 4 in 48 (tens place), and the 2 will be held.

Then $3 \times 4 = 12$, and $12 + 2$ (held) = 14. The 14 is one place higher than the 4, or hundreds. Why? So the 4 in 14 is recorded on the left of tens in hundreds place, and the 1 on the left of hundreds in thousands place. Adding the partial products you get 1,536.

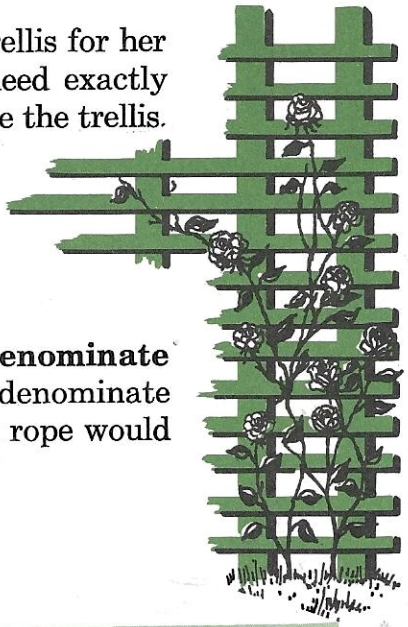
This process is sometimes called the **general method** in multiplication.

Multiply, using the general method:

18	26	36	64	80	54	55	18
12	14	26	15	18	34	24	56

Multiplying Measurements in Different Units

Dave's mother asked him to build her a trellis for her climbing roses. Dave found that he would need exactly 14 pieces, each 6 ft. 3 in. long, in order to make the trellis. How many feet of wood must Dave order?



$$\begin{array}{r} 6 \text{ ft. } 3 \text{ in.} \\ \times\ 14 \\ \hline 84 \text{ ft. } 42 \text{ in.} = 87 \text{ ft. } 6 \text{ in.} \end{array}$$

Numbers like 87 ft. are sometimes called **denominate** numbers. There are several different kinds of denominate numbers. 3 yd. 2 ft. 6 in. is one. How much rope would be needed for 4 lengths of this measurement?

$$\begin{array}{r} 3 \text{ yd. } 2 \text{ ft. } 6 \text{ in.} \\ \times\ 4 \\ \hline 12 \text{ yd. } 8 \text{ ft. } 24 \text{ in.} = 15 \text{ yd. } 1 \text{ ft.} \end{array}$$

Why?

1. How many feet are there in 1 furlong? From the table you can see that there are 40 rd. in 1 furlong, and you can also see that there are $16\frac{1}{2}$ ft. in one rod. You can then find out how many feet there are in 1 furlong by multiplying the number of feet in a rod by the number of rods in a furlong. You can do the multiplication as shown, or you can change $16\frac{1}{2}$ ft. to 16 ft. 6 in.

Table of Measures

12 inches (in.)	= 1 foot (ft.)
3 ft.	= 1 yard (yd.)
$16\frac{1}{2}$ ft.	= 1 rod (rd.)
$5\frac{1}{2}$ yd.	= 1 rd.
40 rd.	= 1 furlong (fur.)
320 rd.	= 1 mile (mi.)
5280 ft.	= 1 mi.

$$\begin{array}{r} 16\frac{1}{2} \\ \times\ 40 \\ \hline \end{array}$$

2. How many rods are there in $3\frac{1}{2}$ miles? What are the two ways of doing this multiplication?

3. How many feet are there in 8 miles?

4. Multiply:

a.	**b.**	**c.**
7 rd. 3 yd. 2 ft.	1 mi. 3 fur. 17 rd.	4 hr. 32 min. 2 sec.
11	6	5

Practice for Skill

Multiply:

	a.	b.	c.	d.	e.	f.	g.	h.
1.	23 × 2	12 × 4	40 × 2	15 × 3	41 × 5	64 × 2	31 × 6	22 × 3
2.	24 × 3	37 × 2	45 × 4	19 × 3	22 × 5	36 × 6	58 × 2	46 × 3
3.	123 × 3	244 × 2	111 × 6	313 × 3	242 × 2	433 × 3	234 × 2	512 × 4
4.	324 × 4	245 × 2	326 × 3	448 × 2	534 × 5	476 × 3	545 × 4	638 × 3
5.	22 × 33	44 × 21	31 × 23	42 × 12	54 × 21	62 × 43	21 × 56	41 × 67
6.	36 × 25	48 × 32	67 × 29	34 × 46	27 × 68	73 × 49	85 × 53	67 × 29
7.	321 × 34	446 × 22	203 × 13	124 × 22	323 × 31	412 × 42	632 × 23	521 × 34
8.	216 × 35	423 × 44	135 × 12	607 × 56	347 × 73	594 × 68	787 × 36	248 × 94

9. 27 yd. 1 ft. 5 in. × 2

7 yd. 1 ft. 4 in. × 4

5 yd. 2 ft. 9 in. × 3

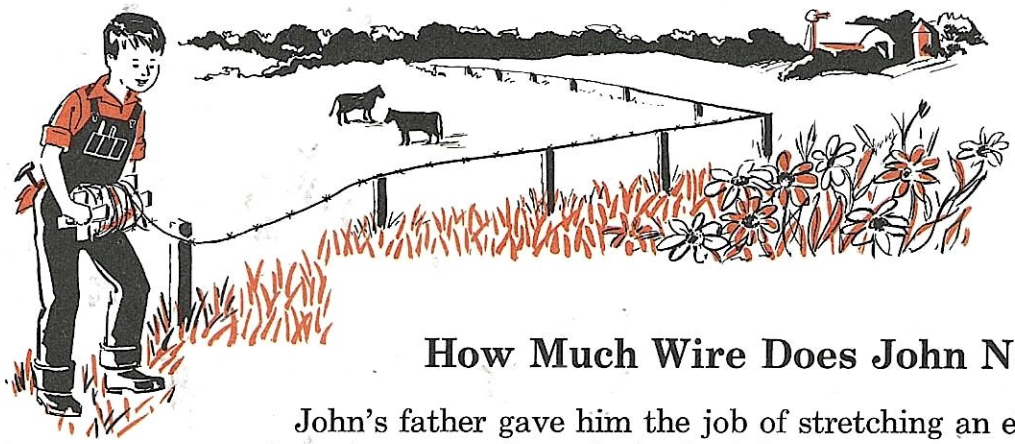
10. 14 rd. 3 yd. 1 ft. × 9

8 mi. 3 fur. 7 rd. × 6

27 rd. 5 yd. 2 ft. × 5

11. 5 gal. 3 qt. 1 pt. × 4

2 pints (pt.) = 1 quart (qt.)
4 qt. = 1 gallon (gal.)



How Much Wire Does John Need?

John's father gave him the job of stretching an electrified wire across the end of a field that was to be used as a cattle pasture. He knew that there were 33 ft. between the posts, and that there were 42 of these spaces. What length of wire must he have for the total distance?

33 ft.
× 42

If you multiply by using tens and ones in the numbers, you must take 3 tens and 3 ones 42 times, or 40 and 2 times. 2 times 3 tens and 3 ones = 66. Why? How must the 66 be recorded as the first partial product?

```
  33
× 42
  66
1320
1386
```

Next, you think:

3 tens + 3 ones
× 40
120 tens + 120 ones

40 times 3 tens and 3 ones = ? You get 40 × 3 by thinking 10 × 3 = 30. Then 4 × 30 = 120. Why? 120 tens + 120 ones = 132 tens + 0 ones. Why? 132 tens + 0 ones = 13 hundreds + 2 tens + 0 ones = 1 thousand + 3 hundreds + 2 tens + 0 ones. Why? Then your second partial product is 1,320.

Adding the partial products, you find that John needed 1,386 ft. of wire.

Is it necessary to record the zero in ones place in the second partial product, 1,320?

1. Multiply:

27	53	41	76	82	38	63	45
16	26	38	23	16	24	27	34

Hal Paints the Picket Fence

Hal had 28 sections of picket fence that needed paint. There were 20 pickets in each section. Jane wanted to know how many pickets there were in all. Hal told her to multiply 28×20.

$$\begin{array}{r} 20 \\ \times\ 28 \\ \hline 160 \\ 40 \\ \hline 560 \end{array}$$

Using the general method, she would have $8 \times 0 = 0$, or 0 in ones place; $8 \times 2 = 16$, or 6 in tens place and 1 in hundreds place in the first partial product. Explain these values.

Then $2 \times 0 = 0$. The 0 is recorded in tens place. $2 \times 2 = 4$. The 4 is recorded in hundreds place in the second partial product. Why?

Hal has 560 pickets to paint.

How would you multiply if Hal had 20 sections of 28 pickets each?

$$\begin{array}{r} 28 \\ \times\ 20 \\ \hline 00 \\ 56 \\ \hline 560 \end{array}$$

In $0 \times 8 = ?$, you would have to think zero or not any of anything gives not any, or $0 \times 8 = 0$ for ones place. Then $0 \times 2 = 0$ for tens place. Now by the general method you could multiply 28 by the 2 in tens place.

Could you shorten the work in this last exercise this way:

$$\begin{array}{r} 28 \\ \times\ 20 \\ \hline 560 \end{array}$$

Because you have ones and tens in 28 and must multiply them by 0 in ones place and 2 in tens place, you know that you will get 0 ones and some tens in the product. Therefore, in work where you multiply by a 0 in ones place, you can write 0 at once in this place in the product and then multiply to find the other values.

Multiply:

26	32	30	28	56	60	35	64
20	40	28	30	10	20	40	20

Interchanging Multiplicand and Multiplier

Which is easier to multiply, $\begin{array}{r} 9 \\ \times\ 19 \end{array}$ or $\begin{array}{r} 19 \\ \times\ 9 \end{array}$? If you think that 9×19 is easier, even though a problem calls for 19×9 ft., will either multiplication give the right answer? Why?

You know from old learnings why you can make these changes in multiplications. For example, 6×8 ft. does not mean the same as 8×6 ft., but if you overlook the measurements and multiply:

$$6 \times 8 = 8 \times 6 = 48 \quad \text{in numbers.}$$

Then, knowing that the answer must be expressed in feet, you can label it accordingly and have the correct answer.

1. Suppose you want to find 16 lengths of 4 ft. each. The problem is 16×4 ft. Can you multiply 4×16 and express your answer in feet? Explain.

2. Will the products in these exercises be equal? Why?

a.

$$\begin{array}{r} 24 \\ \times\ 156 \end{array} \qquad \begin{array}{r} 156 \\ \times\ 24 \end{array}$$

b.

$$\begin{array}{r} 8 \\ \times\ 346 \end{array} \qquad \begin{array}{r} 346 \\ \times\ 8 \end{array}$$

Is it easier to multiply with the larger number in the multiplicand, rather than in the multiplier?

3. Which is the easier of these multiplications? Tell why.

$$\begin{array}{r} 42 \\ \times\ 20 \end{array} \text{ or } \begin{array}{r} 20 \\ \times\ 42 \end{array} \qquad \begin{array}{r} 36 \\ \times\ 150 \end{array} \text{ or } \begin{array}{r} 150 \\ \times\ 36 \end{array}$$

4. Choose the easier way for you to find the products in these multiplications:

$$\begin{array}{r} 8 \\ \times\ 24 \end{array} \quad \begin{array}{r} 7 \\ \times\ 19 \end{array} \quad \begin{array}{r} 95 \\ \times\ 120 \end{array} \quad \begin{array}{r} 13 \\ \times\ 96 \end{array} \quad \begin{array}{r} 9 \\ \times\ 158 \end{array} \quad \begin{array}{r} 212 \\ \times\ 94 \end{array} \quad \begin{array}{r} 218 \\ \times\ 20 \end{array}$$

Checking Multiplication

You know that multiplication is a short method of addition. Can you check multiplication by using addition?

$$\begin{array}{r} 148 \\ \times\ 4 \\ \hline \end{array}$$

When you multiply 148 by 4, you are putting four 148's together like this:

$$\begin{array}{r} 148 \\ 148 \\ 148 \\ +\ 148 \\ \hline \end{array}$$

This method is satisfactory when your multiplier is small. Would you like to check this multiplication problem by addition? Why not?

$$\begin{array}{r} 1763 \\ \times\ 984 \\ \hline \end{array}$$

Another way to check multiplication is to interchange the multiplicand and multiplier.

You can check $\begin{array}{r} 38 \text{ ft.} \\ \times\ 14 \\ \hline \end{array}$ by using $\begin{array}{r} 14 \\ \times\ 38 \\ \hline \end{array}$ if you choose, so long as the answer is known to be feet.

$$\begin{array}{r} 12 \\ \times\ 4 \\ \hline \end{array}$$

A third way to check multiplication is by division. When you check by division, the product becomes the dividend, and the multiplier becomes the divisor.

$$\begin{array}{r} 12 \\ 4\overline{)48} \\ 4 \\ \hline 8 \\ 8 \\ \hline \end{array}$$

Some people check multiplication by redoing their work in the same way. This is not a good method because it is easy to repeat a mistake.

1. Check these multiplications by addition:

a.	**b.**	**c.**	**d.**	**e.**	**f.**
36	49	56	79	126	256
$\times\ 4$	$\times\ 2$	$\times\ 3$	$\times\ 4$	$\times\ 4$	$\times\ 5$

2. Check the following by interchanging the multiplier and the multiplicand:

a.	**b.**	**c.**	**d.**	**e.**
56 tons	32 yd.	39 bbl.	53 min.	42 rd.
$\times\ 23$	$\times\ 41$	$\times\ 27$	$\times\ 16$	$\times\ 20$

Checking Your Understanding

Let's check to be sure that you understand the multiplication 24×36:

$$\begin{array}{r} 36 \\ \times\ 24 \\ \hline 144 \\ 72 \\ \hline 864 \end{array}$$

a. $4 \times 6 = 24$. The 4 is recorded in the same place as the 6. Why? Hold 2. Why?
b. $4 \times 3 = 12$; $12 + 2 = 14$. Why are the 4 and 1 recorded as shown?
c. $2 \times 6 = 12$. Record 2 in one place higher than the 6 being multiplied. Why?
d. $2 \times 3 = 6$; $6 + 1 = 7$. What place values are these? Why?
e. Add the partial products. The product is 864. Why?

a. When multiplying by a number in ones place, the units are the same as those being multiplied. Can you say then that ones times ones give ones, ones times tens give tens, and so on?

b. When multiplying by a number in tens place, the units in the number being multiplied are changed to one place higher. Can you say then that tens times ones give tens, tens times tens give hundreds, and so on?

Find the products:

1.	36	24	36	40	32	18	23	30
	18	26	22	27	19	24	23	28
2.	184	237	94	57	80	104	100	24
	5	8	16	32	24	12	13	35

3. Tell why the work in these exercises is correct but might lead to mistakes:

$$\begin{array}{r} 46 \\ \times\ 18 \\ \hline 46 \\ 368 \\ \hline 828 \end{array} \qquad \begin{array}{r} 72 \\ \times\ 5 \\ \hline 36 \end{array} \qquad \begin{array}{r} 32 \\ \times\ 15 \\ \hline 16 \\ 32 \\ \hline 480 \end{array}$$

4. How could you check a multiplication exercise like 25×36?

A Page of Problems and Exercises

1. If one large box of oranges weighs 63 pounds, how much will 6 boxes weigh?

2. If chickens sell at 54¢ a pound, how much will Ann's mother pay for one that weighs 8 pounds?

3. A rod is equal to $16\frac{1}{2}$ feet. How many feet of fencing wire does Joe need for a square chicken pen that is 2 rods on each side?

4. What is the total length of 8 pieces of wood each 14 ft. long?

5. Mary and her friends want streamers of ribbon for decoration. If each piece of ribbon is to be 18 inches long, how much is needed for 9 pieces of red ribbon? for 8 pieces of blue ribbon?

Use the generalized method:

1. John measures the school yard by pacing. If each step he takes is 28 inches long, what distance does he pace off in 8 steps? in 9 steps?

2. Which is longer, 7 steps of 28 inches or 6 steps of 32 inches? How much longer?

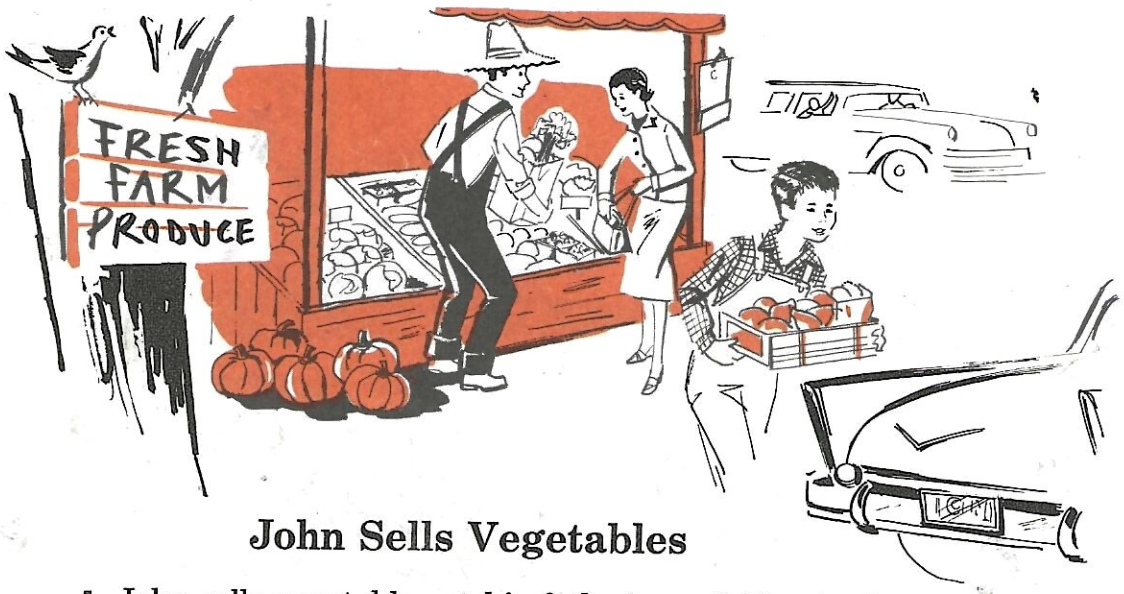
3. Sue wants a spring flower bed of tulips. How many bulbs will it take to plant 7 rows with 24 tulips in each row?

4. In planting bulbs, Joan didn't know which arrangement she wanted: 4 rows of tulips and hyacinths with 36 in a row, or 6 rows with 28 in a row. Which garden would take less bulbs? How many less?

5. Find these products:

a.	**b.**	**c.**	**d.**	**e.**
9	23	128	2 ft. 3 in.	3 yd. 2 ft.
× 7	× 12	× 4	× 4	× 5

f.	**g.**	**h.**	**i.**
672	673	3 yd. 2 ft. 6 in.	6 yd. 2 ft. 6 in.
× 2	× 4	× 3	× 2



John Sells Vegetables

1. John sells vegetables at his father's roadside stand. Will he receive more money by selling 25 large melons at 75 cents each, or 40 small melons at 40 cents each? How much more?

2. If 30 cars pass by the market every hour, how many cars would go by in 6 days for 18 hours a day?

3. About 10 strawberries in every quart basket were soft and had to be thrown away. If 30 boxes were examined, about how many berries had to be thrown away?

4. Another boy works 12 hours a week at the stand. How many hours will he have worked in 22 weeks?

5. A woman offered to buy all the cucumbers for pickling; she would pay 3 cents a pound for all of them. If there were fourteen 12-pound boxes of them, how much did she pay for them?

6. Paper bags cost 50 cents a hundred. If 950 bags were bought, how much did they cost?

7. A dozen ears of corn cost 60 cents. If 15 doz. were sold in one day, how much money was received for them?

8. The average sale for each car that stopped was $1.40. If 10 cars stopped a day, what was the average amount of sales for 7 days?

Connie Learns the Value of Money

$$\begin{array}{r} 25¢ \\ \times\ 17 \\ \hline \end{array}$$

Connie has saved $10.00. She wants to buy 17 articles which cost 25¢ each. Does she have enough money to pay for all of them?

How would you find the answer?

This is the way she figures:

$$\begin{array}{r} \$.25 \\ \times\ 17 \\ \hline 175 \\ 25 \\ \hline \$4.25 \end{array}$$

a. $7 \times 5 = 35$; write 5 in ones place, and hold the 3. The 3 is tens. Why?

b. $7 \times 2 = 14$; $14 + 3 = 17$. Why is the 3 added? Why is 7 put in tens place and 1 in hundreds place?

c. $1 \times 5 = 5$. Write the 5 in what place? Why?

d. $1 \times 2 = 2$. Where is this value recorded? Why?

e. $10.00 − $4.25 = ?, the amount she would have left.

1. Connie runs errands for the lady next door. She is paid $.65 a week. How much does she earn in 14 weeks?

2. Connie had a party for 8 friends. Her father gave her $4.50 to spend. If she spent 21¢ each for 9 favors and 6¢ each for 9 party hats, how much did she spend in all? How much did she have left?

3. Connie did most of the shopping for her mother during the summer. One week she bought a 14-pound ham at 74¢ a pound. What was the cost of the ham?

4. Connie did some baby-sitting. She charged 65¢ an evening. If she sat 12 evenings in May, how much did she earn?

Ways to Earn Money

1. Edward cut grass 3 hours for 75¢ per hour. How much did he earn altogether?

$.75
× 3
5 (complete the work)

2. Alice baby-sat with Mrs. Curry's little girl, Peggy, while Mrs. Curry went shopping. Mrs. Curry was gone 4 hours. What is the amount Alice received if Mrs. Curry paid her $.35 an hour?

3. Judy decided that she would earn money by making punch and cookies. She would sell a glass of punch and a cookie for $.12. One day she had 56 customers. How much money did she receive for the punch and cookies?

4. Johnny and Mary set up a lemonade stand. They decided that they would sell lemonade at 7¢ per glass. One Saturday they sold 76 glasses. What amount did they receive from the sale of their lemonade?

5. Frank has a shoe shine stand. He charges 10¢ per shine. If he had 26 customers, how much did he earn?

6. Gloria's mother helped her make ice cream to sell on their lawn Wednesday afternoon. She sold 17 plates at 10¢ each. If she collected the money as she sold the ice cream, what should her total collection be?

7. Marie's hobby is cooking. She decided to make several blocks of fudge candy. All 27 members of her class at school agreed to buy 2 pieces of fudge at 5¢ each. How much money did she receive?

8. Jane's class chose a project of buying and selling balloons to earn money for a picnic. If they could buy the balloons for 8¢ and sell them for 10¢ each, what would be the profit on 10 dozen balloons?

A Problem Test

1. Lois bought 5 small trays for 79¢ each. What was the total price for the trays?

2. Mr. Willis, the barber, cuts schoolchildren's hair for 75¢. 34 of the boys had their hair cut last week. How much money did Mr. Willis earn for this work?

3. Some of the boys and girls attended a movie twice in one week, and some attended three times. During one week 75 students attended the National Theater twice at 39¢ each. Find the total amount spent for admissions.

4. Sally bought a total of 3 cold drinks a day at 8¢ each. What amount did she spend for cold drinks in 7 days?

5. Jim was responsible for paying the rent for the scout camp group. It cost $3.50 per person weekly. There were 27 boys in Jim's group. How much did he collect for 1 week?

6. Lois, Mary, and Alice each bought 5 small souvenirs. There was a 3-cent tax on each of the souvenirs. What was the total amount of tax paid?

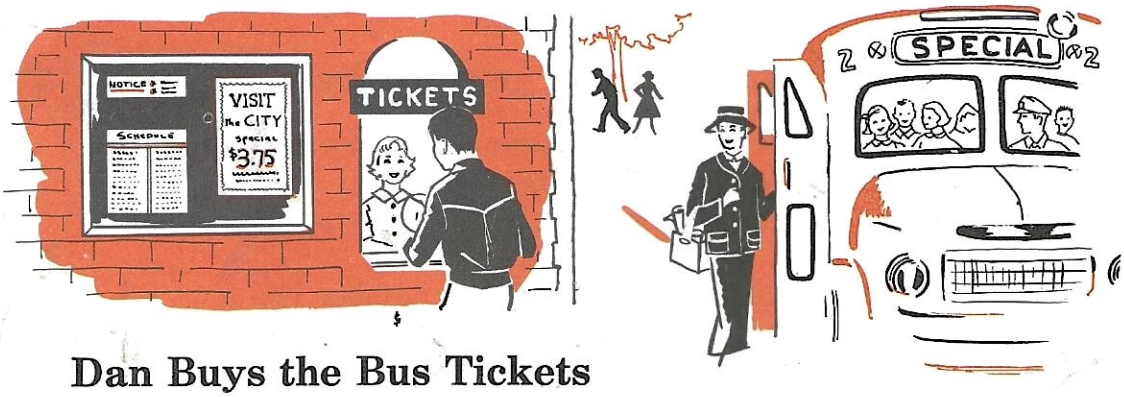
7. Nan had 3 pies which she cut into 4 pieces each. Did she have enough pieces to serve 13 people?

8. Eleven members of the football team each scored 6 touchdowns during the season. How many touchdowns in all were made by these players?

9. 110 people attended a band concert. The lemonade stand sold 48 glasses of lemonade at 12¢ a glass. How much money was obtained from sales at the stand?

10. Ted makes 4 trips on his bicycle between home and school each day. How many trips does he make in 20 days?

11. John is stringing 10 rods of barbed-wire fencing for his father's fence. How much wire (measured in feet) will he need if the fence is to contain 3 strands (different wires)?



Dan Buys the Bus Tickets

Some of the children in the Pinehurst School planned to take a trip with their teacher to the zoo in a nearby city. There were 25 persons who wanted to go.

Dan was chosen to collect the money and buy the bus tickets. The fare for each person was $3.75. After the money was collected, Dan decided that he had better check the amount. First, he counted it in dollars and cents. Then he multiplied one fare by 25. By both methods he got $93.75.

Check his answer:

Use money values for each place in $3.75 and the parts 20 and 5 for 25:

$3.75	3 dollars + 7 dimes + 5 cents	
× 25	× 25	
18 75	18 dollars + 7 dimes + 5 cents	Why?
75 00	75 dollars + 0 dimes + 0 cents	Why?
$93.75	93 dollars + 7 dimes + 5 cents	Why?

$3.75
× 25
18 75
75 0
$93.75

The general method:

Explain each partial product, telling what happens to the money values of the places when they are multiplied first by 5, then by 2.

Multiply:

$4.75	$2.64	$12.64	$2.50	$6.25	$4.75	$3.78	$4.06
2	6	4	8	13	24	40	24

Multiplying Larger Numbers

You should be able to multiply three-place numbers by two-place numbers by the general method.

The things to remember are:

a. the names of the places.

b. what happens to the units in the number being multiplied.

1. What units are 10 times as large as hundreds?

2. What place is next higher than thousands? next larger than ten-thousands?

3. Tell how each place below millions is related to that place.

4. Multiply 36×264:

$$\begin{array}{r} 264 \\ \times\ 36 \\ \hline 1584 \\ 792 \\ \hline 9504 \end{array}$$

a. How did you get 1 in thousands place in the first partial product?

b. Why is the 2 placed in tens place in the second partial product?

c. Explain how 9 hundreds and 7 thousands are obtained in the second partial product.

5. You understand multiplication if you can answer these questions:

	4	3	2
	×	2	3
		9	
	6		
9	9	3	6

a. How do you obtain 9 in tens place?

b. What multiplication gives 6 in hundreds place?

c. Where should you place the 12 that you get from 3×4?

d. How do you know that there are going to be thousands in the product before you even start multiplying?

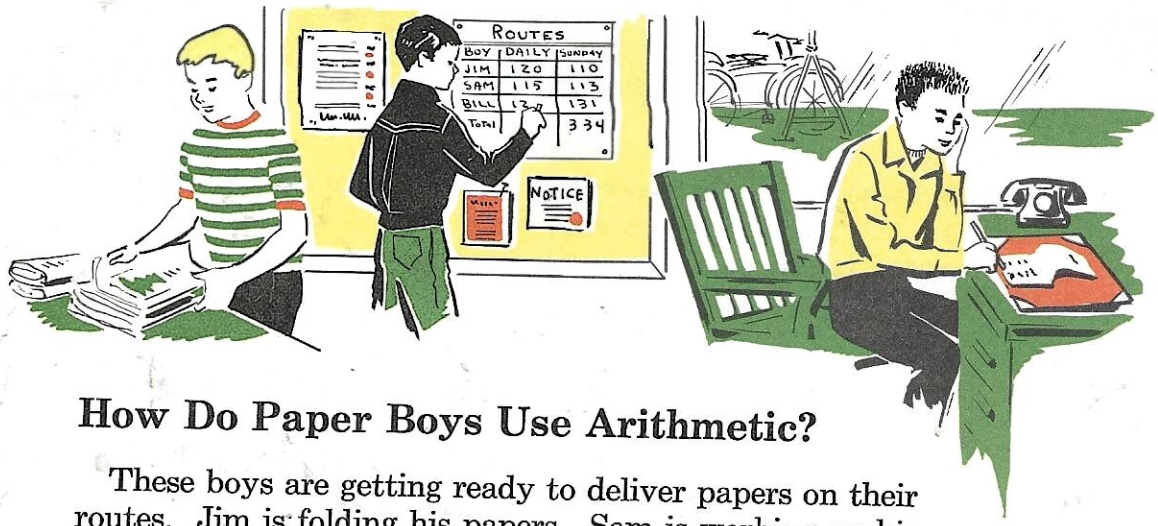
e. Can you put a zero (0) in ones place in the second partial product? Why?

f. Fill the blank spaces in the partial products.

6. Multiply:

235	134	422	514	453	235	336	436	527
42	51	36	27	38	67	14	54	63

3-Place Multiplicands



How Do Paper Boys Use Arithmetic?

These boys are getting ready to deliver papers on their routes. Jim is folding his papers. Sam is working on his records. What records should a paper boy keep? Why?

Bill is writing the number of deliveries he made yesterday, so the big chart can be kept up to date.

Would each boy know his profit for one paper and the number of papers he sold during one day? Would he multiply these figures together to get his daily profit?

1. Bill, who makes 2¢ on each paper, wants to know how much he earns each day if he delivers 123 papers daily.

Are these figures correct?

meanings:

$$\begin{array}{r} 2¢ \\ \times\ 123 \\ \hline 6 \\ 40 \\ 200 \\ \hline 246 \end{array}$$ or \$2.46

or

general method:

$$\begin{array}{r} 2¢ \\ 123 \\ \hline 246 \end{array}$$

or \$2.46

Can the answer be found by interchanging the numbers? Tell what you must remember if you use this easier method.

2. Multiply, first as the exercises are given; then interchange the numbers:

4	6	5	8	3	4	6
111	120	124	112	133	222	333

Practice in Multiplication

Use the general method:

1. The manager of the newspaper agency wanted to know how many daily papers Jim, Sam, and Bill delivered in one year. If the boys delivered a daily total of 358 papers for 312 days each year, what was their yearly total?

$$\begin{array}{r} 358 \\ \times\ 312 \\ \hline \end{array}$$

$$\begin{array}{r} 358 \\ \times\ 312 \\ \hline 716 \\ 3\,58 \\ 107\,4 \\ \hline 111{,}696 \end{array}$$

Do you know all the steps in the general method?

a. When you multiply by a figure in ones place, the units first obtained are the same as those being multiplied. Are any changes ever made? When?

b. When you multiply by a figure in tens place, the units being multiplied are raised ? places. Why?

c. When you multiply by a figure in hundreds place, the units being multiplied are raised ? places. Why?

2. Find how many papers would be sold in 312 days if the daily sale was 420 papers.

Multiply, first as the exercises are given, then interchange the numbers:

3.	**a.**	**b.**	**c.**	**d.**	**e.**	**f.**	**g.**	**h.**
	28	32	56	48	64	72	84	20
	156	222	111	120	132	125	105	116

Find the products:

4.								
	302	119	200	546	678	823	576	405
	20	56	19	12	14	27	25	26
5.	489	436	617	711	444	666	720	809
	257	512	508	233	444	555	160	214
6.	5264	7382	6058	4565	2605	3784	3210	5736
	738	450	219	376	574	306	140	225

Straight Thinking Needed!

What information is needed to make these into problems that can be solved?

1. Judy wants to find out how many bales of hay her pony will need for the winter.

2. Look at the picture at the beginning of this unit. Bob wants to find out how many corn shocks are in his field without counting them one at a time. Can you tell him?

3. Mary, who lives in Ocean City, is going to visit Susy, who lives in Lost Cavern. Can you find these place names in the picture at the beginning of the unit? How far must Mary travel?

4. Art needs to know how many bundles of asphalt shingles he'll need to reroof his barn.

5. The girl scouts have orders for 134 dozen cookies. After the sale of the cookies how much more money will they need to buy a new tent?

6. How can Carl find how many bushels of potatoes his farm is likely to yield?

Find the answers:

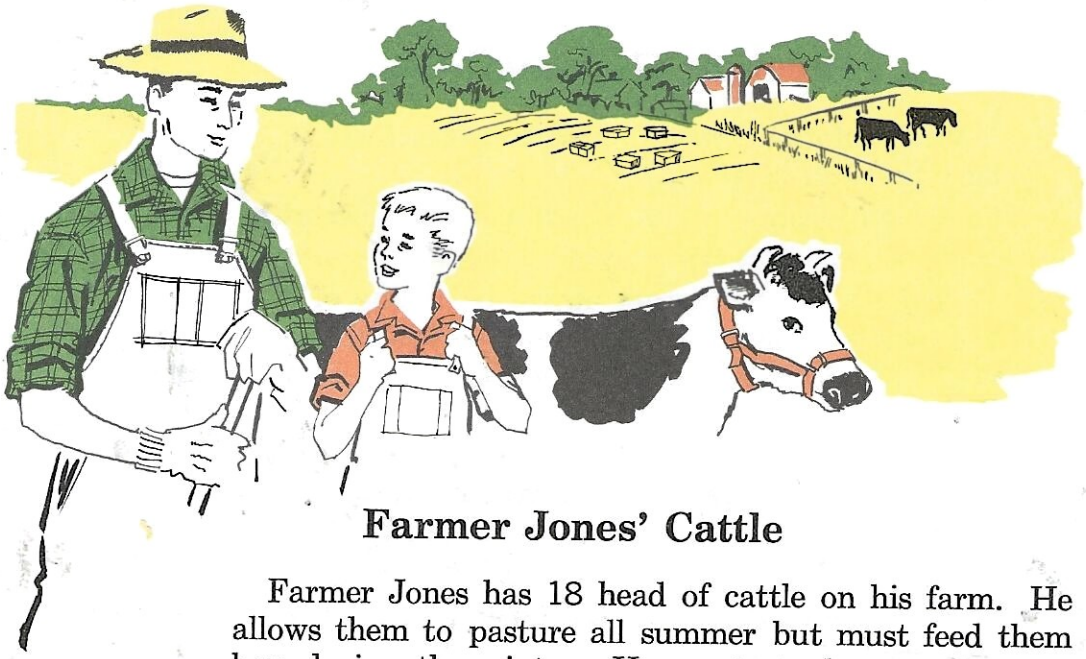
7. A nursery has planted 2,750 pine tree seedlings each year for the past 7 years. How many trees is this in all?

8. The city aquarium has 236 tanks of fish. Each tank contains 36 fish. How many fish does the aquarium own?

9. If each coal car holds 35 tons of coal, how much coal is there in a train of 83 coal cars?

10. Tomatoes are planted 15 inches apart. Jim has 50 plants to set out. How long a row will he need for this many plants? (Remember, there are 49 spaces.)

11. About 24 inches of string are needed for tying up a tomato plant. If there are 60 plants, will a ball containing 100 feet of string be enough?



Farmer Jones' Cattle

Farmer Jones has 18 head of cattle on his farm. He allows them to pasture all summer but must feed them hay during the winter. He wants to be sure to store enough hay, but he also wants as much land as he can spare to grow a crop of wheat. His son, Joe, can approximate the amount he needs:

Joe knows that the cattle ate 5 bales of hay each day last year for 6 months.

1. About how many days will his animals be in the barn?

2. What will be Joe's estimate of the number of bales needed for the winter months?

3. Would he have enough hay if he planted an 18-acre field and each acre yielded 60 bales?

It is always a good idea to check the actual product of any multiplication by comparing it with an estimated product.

4. Estimate the product of 32×48.

Is 30×50 a fair estimate? $30 \times 50 = ?$ (Do the estimating in your head.)

Estimate; then do the work to find the products:

5.	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**
20	49	25	268	150	999	58
$\times$ 103	$\times$ 285	$\times$ 614	$\times$ 124	$\times$ 120	$\times$ 12	$\times$ 62

Review Problems

1. There were 38 children in Miss Jones' class planning to visit several museums in a city 60 miles from their home. Each person had to pay $1.53 for a ticket. How much did the entire group of children pay?

2. If Mrs. Snow needed 15 yards of rope to make 4 lengths of clothesline for the back yard, how much would this cost her at 2¢ a foot?

3. Twelve girl scouts bought dungarees to wear at camp. What was the total cost of all the dungarees at $2.98 each?

4. Why is this correct?

$$\begin{array}{r} 176 \\ \times\ 25 \end{array} = \begin{array}{r} 1 \text{ hundred} + 7 \text{ tens} + 6 \text{ ones} \\ \times\ 25 \\ \hline 25 \text{ hundreds} + 175 \text{ tens} + 150 \text{ ones} = 4400 \end{array}$$

5. A box of 300 disposable tissues costs 28 cents. A box of 200 costs 14 cents. How much would you have to pay for 13 of the larger boxes?

6. The boy scouts collected scrap paper to earn money for a troop flag. 23 boys worked for 9 hours on Saturday. They collected 273 hundred pounds for which they were paid 18 cents a hundred. How much did they earn altogether?

7. The girl scouts held a white elephant sale from 7 until 10 on Friday night. 83 people attended. They sold 87 of the twenty-five cent items. How much money did the girls take in from these?

8. Jackknives for boys cost $1.10 each. William needs two knives for camp. How much money does he need in order to buy them?

9. Using whole numbers, which of the three would be the largest, the multiplicand, the multiplier, or the product? Tell why.

Unit Review

1. Do you get ones, tens, hundreds, or thousands units when you multiply the ones in the multiplicand by a figure in ones place in the multiplier? by a figure in tens place? by a figure in hundreds place?

2. What changes are made in tens units if they are multiplied by a figure in tens place?

3. How many places does multiplying by hundreds change any unit? Does it become a higher or lower unit?

4. Explain the use of zeros in these exercises:

$$\begin{array}{r} 32 \\ \times\ 20 \\ \hline 00 \\ 64 \\ \hline 640 \end{array} \qquad \begin{array}{r} 32 \\ \times\ 20 \\ \hline 640 \end{array} \qquad \begin{array}{r} 40 \\ \times\ 20 \\ \hline 800 \end{array}$$

5. Tell why this is a correct way to multiply by 10 (think the place values first, then use the general method):

$$\begin{array}{r} 482 \\ 10 \\ \hline 4820 \end{array}$$

6. Why can multiplying a number by 100 be done by just adding 2 zeros on the right of the number being multiplied? Would multiplying by 1,000 add 3 zeros? Why?

7. If you wanted to multiply a number by 25, why would multiplying by 100 and then dividing by 4 give you the right answer?

8. Multiply:

a.	b.	c.	d.	e.	f.	g.
84	72	44	144	888	960	28
25	25	25	25	25	25	40

9. Multiply:

a.	b.	c.	d.	e.	f.	g.
8	7	36	125	405	16	472
9	12	14	16	120	124	328

Unit Test

1. Joe lives a long distance from the main part of town. Jim helped him grow cantaloupes. Last summer Joe was able to grow and sell 112 cantaloupes. He made 16¢ on each of them. How much did he make altogether?

2. One Saturday Helen worked for the Rotary Club. She sold tickets for a dance at $1.00 each. For each ticket she received 10¢. How much did Helen earn if she sold 23 tickets?

3. Joe and Carl sold 79 magazines for the White Publishing Company, making 8¢ on each. What amount did they earn altogether?

4. Jane picked 11 quarts of cherries. She helped can 6 quarts and sold 5 quarts at 15¢ each. How much money did Jane receive for the 5 quarts?

5. Mr. Brown showed a film to the fifth- and sixth-grade classes. What facts do you need to determine in order to find the number of pupils who actually attended the showing of the film?

6. Blanche decided to earn money by selling her collection of large sea shells. One Thursday afternoon she sold 17 for 11¢ each. How much money did she get from the sale of the shells?

Multiply and check:

7.

a.	**b.**	**c.**	**d.**	**e.**	**f.**	**g.**	**h.**
42	36	127	120	205	26	48	96
8	12	24	28	32	131	120	232

8. Multiply:

8 yd. 1 ft. 8 in.	2 yd. 8 in.	5 ft. 9 in.	27 in.
× 5	× 7	× 4	× 8

9. What is the cost of 148 ft. of fencing at 16¢ a foot?

10. What would Mary have to pay for 8 yards of dress goods at a price of $1.68 a yard?

UNIT 5

Cooperation — Division of Labor

The division of a whole into its parts is always present in our daily lives. The different children in your class are an illustration of the division of a whole into its parts. How?

Sharing is a division process also. How often have you wanted to find the equal parts into which some whole could be divided?

Is the time given to your arithmetic period a result of division? How?

Division is used in different ways when people work and live together. It is called dividing work or the *division of labor*. Have you ever heard this expression before? Do you know what it means? Does playing baseball or football involve division of labor? Does each player have a job to do in a game? You may consider some positions on a team more important than others, but, in one way or another, all positions are important. Why?

Do you have chores to do around the house? Is this a division of labor?

If your class was planning to build a fence around your playground, like the one on the opposite page, would the members have to be divided into labor groups? How?

If there were 64 fence posts to be put up and there were 8 boys to do the work, how many would each boy put up? Do you find the answer by division, or must you say that each boy is going to do his share before you know?

1. Name all the places in the picture on the opposite page where you see that the process of division is needed.

2. Tell how the units used in measurements, as on a yardstick, a tape measure, or a carpenter's rule, are examples of division.

3. Is division related to subtraction? How?

4. Are division and multiplication related processes? Why?

5. Which process, subtraction or multiplication, would you use to check a division? Explain your answer.

Building a Fence

See the fence the children have started to build. Do you think that they have planned and are managing well? Why?

Why is the girl on the left measuring the boards? Can these children tell how many boards they will need before they start building the fence, or will they have to buy them one by one? How could they tell before they start?

1. Suppose the boards were 4 in. wide and the fence was to be 84 ft. long. How many boards would they need for the fence?

2. How can they find how many nails they need?

3. Why don't answers always come out even when you are dividing? Is it important that they should? Why?

4. Do you know what we call the left-over part when the answer does not come out even in the division?

5. Remainders are often divided to give a fraction in a quotient. Why?

6. What is a fraction?

7. Does a fraction ever express that a division is to be done? When?

Place Values in Division

1. Divide by writing out the names of the places:

$$3\overline{)765} = 3\overline{)7 \text{ hundreds} + 6 \text{ tens} + 5 \text{ ones}} = ?$$
$$= 3\overline{)6 \text{ hundreds} + 16 \text{ tens} + 5 \text{ ones}} = ?$$
$$= 3\overline{)6 \text{ hundreds} + 15 \text{ tens} + 15 \text{ ones}} = ?$$

The quotient is ?.

Now do this exercise by thinking place values, and see if you use the same numbers in the different places.

2. The 6 boys in the Eagle Patrol of the scout troop were collecting scrap iron to buy uniforms. If they collected iron worth $27.65 and the money was divided equally, how much would each boy have toward buying his uniform?

3. Carl, Sue, and Cathy helped their mother gather eggs. They sold many of them at the grocery store. Their mother said that they could divide all of the month's income above $50.00 for eggs sold among themselves. The children received $53.44 from sales for the month of December. How much did each child have as his share if the excess was divided equally?

4. On Arbor Day the public schools of a Michigan town were given 976 small pine trees to plant in the park. If each of 4 schools received an equal share of these trees, how many trees did each school plant?

Divide:

	a.	b.	c.	d.	e.
5.	$2\overline{)874}$	$3\overline{)753}$	$4\overline{)552}$	$5\overline{)795}$	$6\overline{)972}$
6.	$4\overline{)408}$	$5\overline{)625}$	$6\overline{)672}$	$7\overline{)903}$	$8\overline{)864}$

Divide and check two ways:

	a.	b.	c.	d.	e.	f.
7.	$3\overline{)393}$	$5\overline{)580}$	$2\overline{)632}$	$4\overline{)124}$	$4\overline{)960}$	$6\overline{)636}$
8.	$6\overline{)474}$	$4\overline{)524}$	$6\overline{)726}$	$8\overline{)688}$	$7\overline{)847}$	$5\overline{)325}$

Building a Bookcase

Jim wants to divide a 72-in. board into 4 equal parts to make shelves for a bookcase.

He can do this by a faster method than writing or thinking place values. Let's see what the method is:

He is dividing 72 in. into 4 equal parts: $4\overline{)72} = ?$ For every place value in 72 there must be a place value in the quotient. Why?

$$\begin{array}{r} 1 \\ 4\overline{)72} \\ \underline{4} \\ 3 \end{array}$$

$7 \div 4$ is the first step. 4 out of the 7 can be used. $4 \div 4 = 1$, the first quotient figure. There is a remainder in this place. Put 1 in the quotient for the place held by the value that was used. (This 1 is really a ten. Why?)

$$\begin{array}{r} 18 \\ 4\overline{)72} \\ \underline{4} \\ 32 \\ \underline{32} \end{array}$$

The unused 3 in the first place and the 2 in the second place give 32 units in this second place. Why?

$32 \div 4 = 8$, the second quotient figure. There is no remainder. Put the 8 in the place held by the 2 in 32. Why?

The quotient is 18 in. for each of the 4 equal parts.

1. Kathy asked her brother, Randy, to make 3 shelves in her playhouse. If the board he has is 66 inches long, how long will each shelf be?

2. Nine girls plan to divide 108 feet of wood to make stilts. How many feet of wood will each girl get?

3. Gary wants to divide 96 ft. of lumber into 6 equal parts. How many feet will there be in each part?

4. How long will each piece be if an 84-inch board is divided into 6 equal pieces?

Zeros in Dividends and Quotients

When any number is being divided, you know that there should be a figure in the quotient for every place in the dividend. Why?

Sometimes you will have zeros in the different places in the quotient. If 0 is found to be the first figure, it is omitted. At any other time it must be included. Why?

1. Suppose that Jim wants to divide 153 ft. into 3 equal parts. What will be the length of each part?

$3\overline{)153 \text{ ft.}} = 3\overline{)1 \text{ hundred} + 5 \text{ tens} + 3 \text{ ones}} = ?$

Will there be a figure in hundreds place in the quotient?

In order to be able to divide, you must change the hundred to tens, like this:

$3\overline{)153} = 3\overline{)0 \text{ hundreds} + 15 \text{ tens} + 3 \text{ ones}}$

Do you put a 0 in hundreds place in the quotient? Why?

$3\overline{)15 \text{ tens} + 3 \text{ ones}} = 5 \text{ tens} + 1 \text{ one} = 51$. Why?

Here is this same division without naming the places:

```
  51
3)153
  15
   3
   3
  ---

 101
8)808
  8
  --
  0
  0
  --
  8
  8
  --
```

Why is the quotient figure of 5 placed in tens place?

Explain the remaining steps of the division. One of the 3 equal parts of 153 ft. is ? ft.

2. If you were dividing 808 by 8, in what place in the quotient would you need to write a zero? Can you say that the quotient is 1 hundred and 1 one?

How do you read the number 101 without naming the places?

3. Divide:

$7\overline{)805} = ?$ $5\overline{)515} = ?$ $9\overline{)918} = ?$ $8\overline{)720} = ?$ $6\overline{)900} = ?$

Checking Your Learning

In the work with the process of division in this unit, the divisor has been a number which told into how many parts some whole was to be divided. In this work you were asked to find the size of one of the equal parts.

You considered place values in the dividend. Divisors received no such consideration. For example, in the division $4\overline{)72}$ in. you do not think: How many groups of 4 ones are there in 7 tens and 2 ones? In this division your question is: What is the size of one part when 72 is divided into 4 equal parts?

Be sure that your thinking is straight in the following:

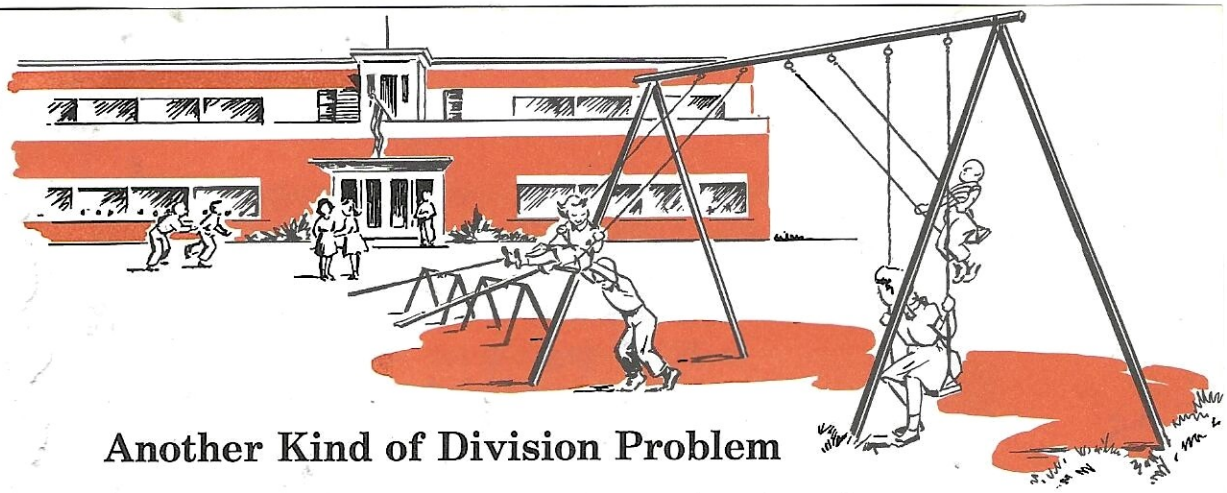
1. Mary Ann gathered 1345 sea shells during her summer at the beach. She planned to share them with 4 girl friends. How many would each girl get?

2. The scout troop was asked to plant 2,328 pine trees during their 6 days in camp. How many should they plant each day if they planted the same number daily?

	a.	b.	c.	d.	e.
3.	$9\overline{)117}$	$4\overline{)2640}$	$3\overline{)4302}$	$2\overline{)2412}$	$7\overline{)6909}$
4.	$6\overline{)4830}$	$8\overline{)8072}$	$4\overline{)8316}$	$5\overline{)9360}$	$3\overline{)3297}$
5.	$7\overline{)364}$	$7\overline{)868}$	$6\overline{)312}$	$8\overline{)496}$	$8\overline{)928}$
6.	$4\overline{)952}$	$3\overline{)735}$	$5\overline{)495}$	$4\overline{)840}$	$4\overline{)908}$
7.	$3\overline{)1041}$	$4\overline{)5052}$	$8\overline{)9000}$	$6\overline{)7008}$	$7\overline{)2905}$

To show that you know the meanings in division, write out the names of the places as you do these:

8.	$2\overline{)2564}$	$6\overline{)798}$	$4\overline{)3336}$	$7\overline{)1197}$	$8\overline{)1024}$



Another Kind of Division Problem

There are times when you know the size of some quantity, and also know that it is a part of a larger quantity. But you may not know how many parts of the smaller size there are in the larger. For example:

For safety on a school playground, swings like those shown in the picture must allow $4\frac{1}{2}$ feet of space for each swing attached to the frame.

1. How many swings should be put on a frame that has an overall length of 15 ft.?

One way to solve such a problem is by taking measurements of $4\frac{1}{2}$ ft. from one end of the frame to the other. This is the same as the subtraction way:

$$15 - 4\tfrac{1}{2} = ?,\ ? - 4\tfrac{1}{2} = ?, \text{ and so on.}$$

Why not use the division: $4\frac{1}{2}\overline{)15} = ?$

Do you think that time can be saved when the numbers are large if division is used? Why?

2. How many groups of 6 apples are in 48 apples?

Subtracting 6 from the remainder each time will tell you how many groups of 6 there are in 48. Why?

```
 48
- 6
 42
- 6
  ?
```

You know the answer to the division by using old multiplication facts. There are 8 groups of 6 apples each in 48 apples.

```
   8
6)48
  48
```

3. How many groups of 9 things each are there in 81 things? in 72 things? in 54 things?

Finding the Number of Groups

Let's think out the meaning of finding how many groups of 6 things each there are in 864 things.

6 things⟌864 things

```
  100
6⟌864
  600
  264
```

If you think, how many groups of 6 ones in 8 hundreds, what does a quotient figure in hundreds place tell you?

It is easier to understand the division if you use 6 ones⟌800 ones. Will there be as many as 100 (or 200) groups of 6 ones? 100 groups will use how many of the 864 ones? Will $8 \div 6 = 1$, and a remainder of 2, tell you that the quotient figure stands for 100 groups, and that you have used 600 of the 864 ones? Do you need to write the two 0's in both the 100 and 600 if you have 1 in hundreds place in the quotient and 6 under the 8 hundreds in the dividend?

```
  4  }
 40  } = 144
100  }
6⟌864
  600
  264
  240
   24
   24
```

Why do you get 4 in tens place and 4 in ones place in the quotient?

Now leave empty spaces where you don't need 0's:

```
 144
6⟌864
  6
  26
  24
   24
   24
```

$8 \div 6$ gives a quotient figure of 1 in the place held by the 8 (hundreds). Why?

$26 \div 6$ gives a quotient figure of 4 in the place held by the 6 (tens). Why?

$24 \div 6$ gives a quotient figure of 4 in the place held by the 4 (ones). Why?

There are 144 groups of 6 things in 864 things.

1. How many measurements of 7 in. each are there in 91 inches? in 364 inches?

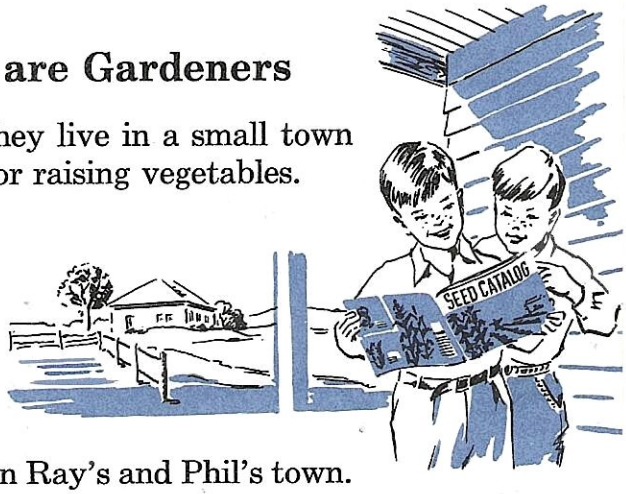
2. What does the first quotient figure in this division mean?

```
   5
8 ft.⟌424 ft.
```

3. Explain the first quotient figure in each of these:

$7\overline{)364}$ $4\overline{)708}$ $8\overline{)760}$ $6\overline{)864}$ $5\overline{)990}$ $4\overline{)648}$ $7\overline{)6020}$

Ray and Phil are Gardeners



Ray and Phil are brothers. They live in a small town where they have plenty of land for raising vegetables.

Here in this picture they are discussing the problems of raising corn to sell in the town. They have decided on how much ground to use, how much corn to grow, and about what price the corn will bring.

1. Corn sells by the dozen ears in Ray's and Phil's town. They want to know how many dozen ears they should have from 3,984 plants, if each plant (or stalk) raises 1 ear. Can you find the answer?

```
12)3984
```

You want to know how many 12's in 3984. How will you find the first quotient figure? Will you consider the 12 as ones or as 1 ten and 2 ones?

Should you think, how many of the 12 ones, or the 1 ten and 2 ones, are there in 3 thousands, or even in 39 hundreds?

Do you think it will be easier if you consider the number of groups of 12 ones in 3600 ones? Why will the answer be 300?

```
  30
 300
12)3984
3600
 384
 360
```

Can you write 36 hundreds for 3600 to show the part of the dividend used? Can you write 3 in hundreds place instead of 300 in the quotient? Why? There are 384 still not used. Finish the division.

```
  332
12)3984
  36
  38
  36
   24
   24
```

Here is the same division without writing the zeros. Why is 36 written under the 39? Why is 36 written under the 38?

How many 12's are there in 24?

Tell why the quotient figures are placed in the positions which they now hold.

Comparing the Two Ideas of Division

Do you understand these ideas of division?

A. Size of each part
Number of parts ⟌ Number of things to be divided

B. Number of parts
Size of each part ⟌ Number of things to be divided

The first idea (A) of division:

Suppose you have 42 dozen eggs (the number of things to be divided) to be *shared* by 14 families (the number of parts into which 42 is to be divided). The answer or quotient will be the **size of the part** for each family.

The second idea (B) of division:

Suppose you have a 52 foot rope (the number to be divided) to be cut into 4 ft. lengths (the size of each part). You want to know how many 4 ft. lengths there will be or the **number of parts.**

Do you see the difference between these two ideas of division? Here are some problems and exercises in which you should first decide which idea of division is called for. Then, you should find the answers, being careful to label them correctly.

1. Mary baked 108 cookies for the Campfire Girls to sell. How many boxes does she need if they are to be packaged 12 to a box?

2. Billy's father brought 56 buckets of sand from the beach for the 4 kindergarten sand boxes. If he divides them equally, how many buckets will he put in each box?

3. Which of these divisions is type A? type B? Tell why.

a. 12 eggs ⟌ 36 dozen eggs

b. 8 ⟌ 5280 ft.

c. 3 ft. ⟌ 5280 ft.

d. 5 days ⟌ 1 year

e. 10 rods ⟌ 360 rods

f. 8 ⟌ 4096 hours

One Method in Division

Both kinds of division problems can be done by one method:

a. If you divide 72 in. into 6 equal parts, what is the size of one part?

$$\begin{array}{r} 12 \text{ inches} \\ 6\overline{)72} \\ \underline{6} \\ 12 \\ \underline{12} \\ \end{array}$$

By meanings, 7 tens cannot be divided evenly into 6 equal parts. Use 6 tens; 6 tens ÷ 6 = 1 ten. You can get this same quotient figure by thinking: How many 6's are there in 7? The answer will be in the same place as that held by the 7. The answer is 1 ten. Explain how to divide the 12 ones.

b. If you divide 72 in. into groups of 6 in. each, how many groups will there be?

$$\begin{array}{r} 12 \text{ groups} \\ 6\overline{)72} \\ \underline{6} \\ 12 \\ \underline{12} \\ \end{array}$$

By meanings, there is no whole number of groups of 6 ones in 7 tens. But in 6 tens, 6 tens ÷ 6 ones = 1 ten group.

Again, you can get the same quotient figure by thinking: How many 6's are there in 7? The answer will be put in the same place as that held by the 7.

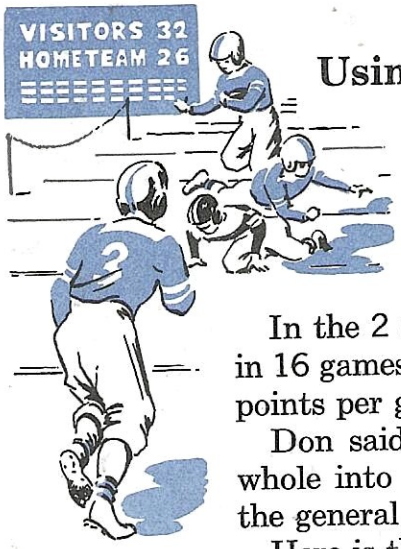
Explain how to divide the 12 ones. Do you see that you have used the same steps in both examples?

This one method is called the **general method** in division. When using the general method, you must be particularly careful to label your answers correctly.

1. How will you label the answers in these divisions?

$6\overline{)144}$ Divide 144 apples into 6 equal parts.	$6\overline{)144}$ How many groups of 6 apples are there in 144 apples?

In these divisions your first step by the general method is: How many 6's are there in 14? Why? Where will the quotient figure be placed? Why?



Using the General Method in Division

Don played quarterback on his school's football team last fall. The team had a successful season. In fact, the year before last when Don played quarterback, the school had a good team.

In the 2 seasons, the team scored a total of 304 points in 16 games. Don wanted to know the average number of points per game scored by his team for the two years.

Don said, "This is a problem where I must divide a whole into equal parts: $16\overline{)304}$. I'll do the division by the general method."

Here is the way Don did the work:

$16\overline{)304}$

$$\begin{array}{r} 19 \\ 16\overline{)304} \\ \underline{16} \\ 144 \\ \underline{144} \end{array}$$

The quotient will be a two-place number. Why?

At each step quotient figures will be put in the same places as the number being divided. Tens will give tens and ones will give ones.

Why is Don thinking only of tens and ones when there are hundreds in the dividend?

Do you have to think place values when you divide?

Can you think this way?

How many 16's are there in 3? Answer: Not any.

How many 16's are there in 30? Answer: 1. How many 16's are there in 144? Are there less than 10 ($10 \times 16 = 160$)? Try 9 ($9 \times 16 = 144$). Where is the 9 placed in the quotient? Why?

The team scored an average of 19 points per game.

1. Divide, using Don's way of finding quotient figures:

$42\overline{)294}$ $13\overline{)286}$ $12\overline{)492}$ $23\overline{)1403}$ $32\overline{)1952}$ $30\overline{)1650}$

2. In $21\overline{)1386}$, why is $13 \div 2$ as good an estimate for the first quotient figure as $130 \div 20$? Where will this figure be placed in the quotient? Complete the division.

Quotient Figures Located by Place Values

Let's solve Ray's and Phil's problem on Page 145 by the general method. You first think: Will the quotient be a four-place number? a three-place number? How many 12's are there in 3?

$12\overline{)3984}$

If you think $3 \div 12$, you will have no value in the quotient place that corresponds to the place held by the 3 in the dividend. Why? Then, look at $39 \div 12$. Why?

Are there any 12's in 39?

The first quotient figure is 3. Its place in the quotient corresponds to the 9 in 39. Why?

Will the quotient have 3 figures in it? Why?

There is a remainder of 3 in this first step. How is it obtained?

```
  332
12)3984
  36
  38
  36
   24
   24
```

When the 8 in the dividend is put with the 3, you have $38 \div 12$, which gives another 3 in the quotient for the second place.

Why do you have $24 \div 12$, which gives the last figure in the quotient? This figure is 2. Why?

Do you see that every quotient figure in the above division can be estimated correctly: $39 \div 12$, $38 \div 12$, and $24 \div 12$? When this happens, the estimated quotient figures are said to be true ones.

Divide, thinking place values:

1. How many dozen ears of corn in a total of 492 ears? in 3,768 ears?

2. $22\overline{)484}$ $13\overline{)390}$ $42\overline{)882}$ $11\overline{)396}$ $24\overline{)576}$

3. $32\overline{)9984}$ $12\overline{)4968}$ $23\overline{)5060}$ $53\overline{)11130}$ $33\overline{)76056}$

Divide, using the general method:

4. $11\overline{)3674}$ $13\overline{)3913}$ $22\overline{)5126}$ $34\overline{)6902}$ $45\overline{)9450}$

5. The Browns planned to take a 5,800 mi. trip by automobile next summer. About how many miles a day would they need to travel if they had 20 days in all for the trip?

Practice—The General Method in Division

It is important to know all of the old division facts in order to be successful when dividing larger numbers. These may be difficult to remember:

1. $7\overline{)63} = ?$ $8\overline{)56} = ?$ $8\overline{)72} = ?$ $9\overline{)81} = ?$ $7\overline{)49} = ?$

2. $7\overline{)56} = ?$ $9\overline{)63} = ?$ $9\overline{)72} = ?$ $8\overline{)64} = ?$ $6\overline{)48} = ?$

3. $5\overline{)45} = ?$ $7\overline{)42} = ?$ $8\overline{)48} = ?$ $8\overline{)40} = ?$ $9\overline{)45} = ?$

4. $6\overline{)42} = ?$ $4\overline{)36} = ?$ $7\overline{)35} = ?$ $6\overline{)36} = ?$ $9\overline{)36} = ?$

5. If there were any of these which you did not know, could you use multiplication facts to help find the answers? Tell how multiplication and division facts are related.

(Example: $3\overline{)27} = 9$ $9 \times 3 = ?$ $3 \times 9 = ?$)

6. Use the general method to do this division:

```
 15?          Think: How many 6's in 9?
6)924         Why do we have 32 to divide in the next step?
 6            How many 6's in 32?
 32           How are the two 24's obtained?
 30           What is the last quotient figure?
  24
  24
```

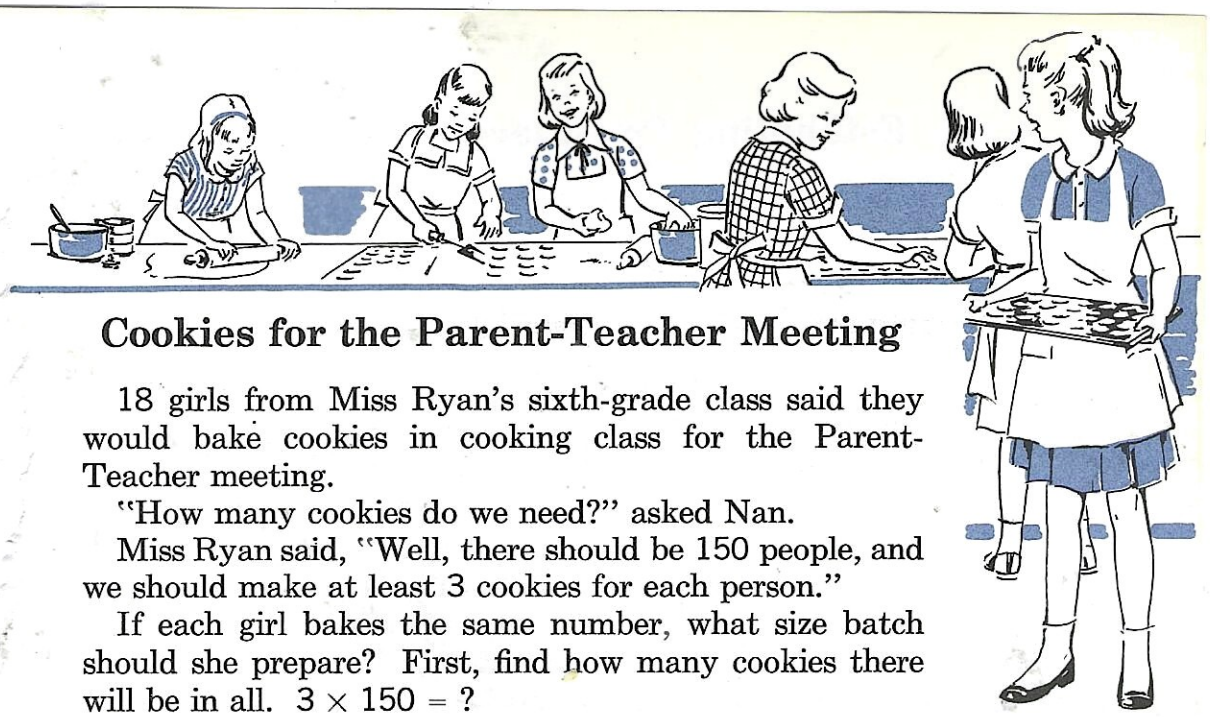
7. Copy and finish these exercises:

18?	12?	2??	2??	???
4)748	7)868	4)996	3)705	5)945
4	7	8	6	?
34	16	19	10	??
32	14	?	?	
2	?			

How do you know that the quotient in each of the above exercises is going to be more than 10? more than 100?

8. Divide:

$2\overline{)752}$ $3\overline{)867}$ $4\overline{)576}$ $5\overline{)915}$ $6\overline{)702}$ $7\overline{)931}$



Cookies for the Parent-Teacher Meeting

18 girls from Miss Ryan's sixth-grade class said they would bake cookies in cooking class for the Parent-Teacher meeting.

"How many cookies do we need?" asked Nan.

Miss Ryan said, "Well, there should be 150 people, and we should make at least 3 cookies for each person."

If each girl bakes the same number, what size batch should she prepare? First, find how many cookies there will be in all. $3 \times 150 = ?$

$18\overline{)450} = ?$

Miss Ryan asked, "When we divide 450 by 18, will the answer be more than 10? more than 20? Will it be a 2-place number?"

Nan said, "To do this problem I would subtract 18's."

Would this process give Nan the right quotient? How?

Tom said, "I estimate the quotient figures and try them. If my estimate is too small, *the remainder will be larger than the divisor*. If my estimate is too large, *I cannot subtract*. In this division I first see that there will be a 2-place quotient. For the first figure, there may be as many 18's in 45 as there are 1's in 4. I write 4 over the 5 and multiply, $4 \times 18 = 72$. 72 is too large."

$$\begin{array}{r} 4 \\ 18\overline{)450} \\ 72 \end{array}$$

Was Tom correct? Why?

$$\begin{array}{r} 3 \\ 18\overline{)450} \\ 54 \end{array} \qquad \begin{array}{r} 2 \\ 18\overline{)450} \\ 36 \end{array}$$

Hal said, "Try 3 or 2."

Which was the correct estimate?

$$\begin{array}{r} 450 \\ -\ 18 \\ \hline 432 \\ -\ 18 \\ \hline \end{array}$$

1. Estimate the first quotient figure by Tom's method:

$16\overline{)496}$ $24\overline{)768}$ $36\overline{)900}$ $15\overline{)465}$ $25\overline{)675}$ $18\overline{)396}$

Estimating Two-Place Quotients

Tom wants to build a fence 85 ft. 4 in. long. If he plans to have fence posts which are placed 32 in. apart, how many posts will he need? (Remember that he will need one more post than the number of spaces.)

85 ft. 4 in. = 1,024 in. Why?

$32\overline{)1024}$ Will the quotient be a 2-place number?

Are there as many as ten 32's in 1,024? ($10 \times 32 = ?$) Are there as many as a hundred? ($100 \times 32 = ?$)

Tom thinks this way: "The answer is a little more than 30. I estimate by letting the 4 in 1024 take care of the 2 in 32. Then, I think of the number of 3's in 102. It's 30 and some left over. So my answer is a 2-figure number."

Tom said, "If I only wanted to find the first quotient figure, I'd think, How many 3's are there in 10, or how many 30's are there in 102?"

Is Tom's method of finding how many 3's in 10 a good way of finding the number of places in the quotient? Is it as good a way as finding the number of 30's there are in 102? Why?

1. Is the quotient of $44\overline{)1408}$ a 2-place number? Why? Is it more than 20? more than 30? less than 40? Tell how you made your estimate.

2. Would you say that the quotient of $63\overline{)3276}$ is a 2-place number? Why? Is it a 50-number? Why?

3. Is the quotient of $75\overline{)1575}$ more than 10? more than 20? Estimate its value.

4. Estimate the quotients in these, telling whether they are a little more or a little less than some tens value:

$19\overline{)361}$	$22\overline{)484}$	$16\overline{)256}$	$15\overline{)225}$	$18\overline{)324}$
$62\overline{)2046}$	$52\overline{)1820}$	$42\overline{)2730}$	$64\overline{)2304}$	$72\overline{)4048}$

Building a Tool House

Ray and Phil wanted a building where they could keep garden tools. Their father told them where to build it.

1. The boys decided to build their tool house with the uprights for the building on 16 in. centers. Do you know what that means? (It is 16 in. from the center of one to the center of the next, either to the left or right.)

Phil wanted to figure the number of uprights that would be needed for a side 10 ft. 8 in. long. He knew that $16\overline{)128}$ would give him the number of spaces and that the number of uprights would be one more than this. Why?

$16\overline{)128}$

$\begin{array}{r} 6 \\ 16\overline{)128} \\ \underline{96} \\ 32 \end{array}$

He noted that both 1 and 12 in 128 would give him 0 in estimating a quotient figure. Thus his answer must be a one-place number. The answer cannot be as much as 10. Why? The figure is found by testing values: How many 16's are there in the number 128?

You can estimate values by first rounding 16 to 20 and 128 to 130. $20\overline{)130} = 6$. Are there six 16's in 128? Trying this value, $6 \times 16 = 96$. Your remainder is more than 16. Thus, your estimate is *false*.

$\begin{array}{r} 7 \\ 16\overline{)128} \\ \underline{112} \\ 16 \end{array}$

Trying 7, $7 \times 16 = 112$.

The remainder is equal to 16. Thus the quotient figure should be 8. Why?

There will be 8 spaces between uprights.

Adding one to this number, Phil will need 9 uprights for this side.

Sometimes rounded numbers will give the correct quotient figures. Do not overlook this method.

Estimating Quotient Figures

1. If Phil wanted to round the numbers in order to estimate the quotient of $32\overline{)1024}$, would he find a true or false first quotient figure?

$32\overline{)1024}$ $30\overline{)1020}$ (rounded to the nearest 10)

How many 30's in 102? Is this work correct?

$$\begin{array}{r} 3 \\ 32\overline{)1024} \\ \underline{96} \end{array}$$

Finish the division.

Does rounding give the true figure?

Would Phil have needed to round the dividend in this exercise? Why?

2. Can Phil estimate the following and get true first quotient figures?

$27\overline{)1620}$	$29\overline{)1160}$	$42\overline{)2940}$	$56\overline{)1680}$	$9\overline{)2560}$	$33\overline{)4950}$
↓	↓	↓	↓	↓	↓
$30\overline{)1600}$	$30\overline{)1200}$	$40\overline{)2900}$	$60\overline{)1700}$	$10\overline{)2600}$	$30\overline{)5000}$

3. If you were estimating how many 48's there are in 288, which would be better thinking: How many 4's are there in 28, or how many 5's are there in 28? Would rounded numbers help you?

4. In estimating the quotient of $56\overline{)280}$, would you try how many 5's are there in 28, or how many 6's are there in 28?

5. Estimate the quotient of $38\overline{)266}$. Is it true or false?

6. Which is the closer estimate for the quotient of $19\overline{)95}$, the number of 1's, or the number of 2's in 9?

7. Tell what you think is the easiest way to estimate the first quotient figure of these, and then tell where it should be placed in the quotient:

$8\overline{)4960}$ $36\overline{)2520}$ $45\overline{)2700}$ $67\overline{)4928}$ $18\overline{)6480}$ $96\overline{)2889}$

Practice Problems and Exercises

1. Al Johnson went with his father on an automobile trip of 1200 miles. If they wanted to stop 4 times during the trip with equal distances between stops, how far would they travel between stops?

2. Mr. Johnson's car weighed 2800 pounds. If this weight were supported equally by all 4 wheels, what was the weight on each wheel?

3. Joe sold greeting cards at 10 cents each. If he sold $100 worth, find the number of cards he sold.

4. Rachel's mother types reports for college students. If she receives 15¢ per page, how many pages must she type in order to earn enough to buy a coat that costs $38.70?

5. Jack digs angleworms for fishermen. If he receives 12¢ per dozen, how many dozen will he have to dig to pay for a $6.84 air rifle?

6. Judy wove rugs out of old rags. It took 9 pounds of rags to make one rug. How many rugs could be made from 6229 pounds of rags?

Find the quotients:

	a.	b.	c.	d.	e.
7.	$22\overline{)5830}$	$36\overline{)5014}$	$19\overline{)8332}$	$27\overline{)9342}$	$24\overline{)6360}$
8.	$17\overline{)5865}$	$32\overline{)7936}$	$28\overline{)9856}$	$14\overline{)840}$	$15\overline{)825}$
9.	$15\overline{)3585}$	$19\overline{)8379}$	$27\overline{)6966}$	$31\overline{)7657}$	$40\overline{)7680}$
10.	$27\overline{)14796}$	$46\overline{)33764}$	$57\overline{)9462}$	$38\overline{)7182}$	$67\overline{)1876}$
11.	$56\overline{)868}$	$49\overline{)7987}$	$17\overline{)7905}$	$45\overline{)5355}$	$31\overline{)8434}$
12.	$13\overline{)390}$	$22\overline{)440}$	$17\overline{)527}$	$24\overline{)480}$	$36\overline{)1440}$

Farmer Green Has Many Problems

1. Farmer Green is showing his son how to plant cabbage plants. He plans to fill a field with them. He first wants to know how many rows 3 ft. apart he can plant in a field that is 2,484 ft. wide. Find the answer for him.

2. How many plants will he be able to plant in each row if they are placed 18 in. apart and the rows are 750 feet long?

3. Look at the fence. It goes around the field. The fence posts are 12 ft. apart. How many posts are there if the distance around the field is 6,480 ft.?

4. As soon as the cabbages are planted, he is going to repair his barn. The first thing he is going to do is put on a new roof. The barn is 90 ft. long. How many shingles will he need to make a row, if the shingles are 30 inches wide?

5. The Greens have a herd of dairy cows. Besides other feed, each cow is given 15 lb. of hay each day. How long will a ton (2,000 lb.) last one cow at this rate?

6. Each cow should have at least 24 sq. ft. of floor space in the barn. If Mr. Green's barn was built with a manger space of 560 sq. ft., how many cows can he handle adequately?

7. Mr. Green sold a total of 454,500 lb. of milk last year. If you figure that each of his 23 cows was equally good, how many pounds of milk did each cow produce?

Naming the Left-Overs in Division

1. Denny was helping his mother put up her new clothesline. She asked him to cut the line into 5 equal parts and then to set some poles to fit the lengths. He measured the line and found that it was 72 feet long. How long will each of the equal pieces be in whole feet? Will there be any left over?

Left-overs in division are called remainders (R). Find the remainders in the following:

	a.	b.	c.	d.	e.	f.
2.	$7\overline{)45}$	$9\overline{)88}$	$3\overline{)76}$	$6\overline{)91}$	$8\overline{)77}$	$7\overline{)85}$
3.	$5\overline{)469}$	$9\overline{)806}$	$4\overline{)375}$	$12\overline{)257}$	$19\overline{)817}$	$16\overline{)300}$
4.	$15\overline{)784}$	$17\overline{)612}$	$14\overline{)1148}$	$17\overline{)1258}$	$16\overline{)1392}$	$24\overline{)4096}$

Do you know how to write the answers when there is a remainder in division?

5.
$$\begin{array}{r} 341 \\ 12\text{ ft.})\overline{4096\text{ ft.}} \\ 36 \\ \hline 49 \\ 48 \\ \hline 16 \\ 12 \\ \hline 4\text{ ft. R} \end{array}$$

If you are finding the number of 12 ft. lengths in a total of 4,096 ft., would you do the work as shown here? Explain.

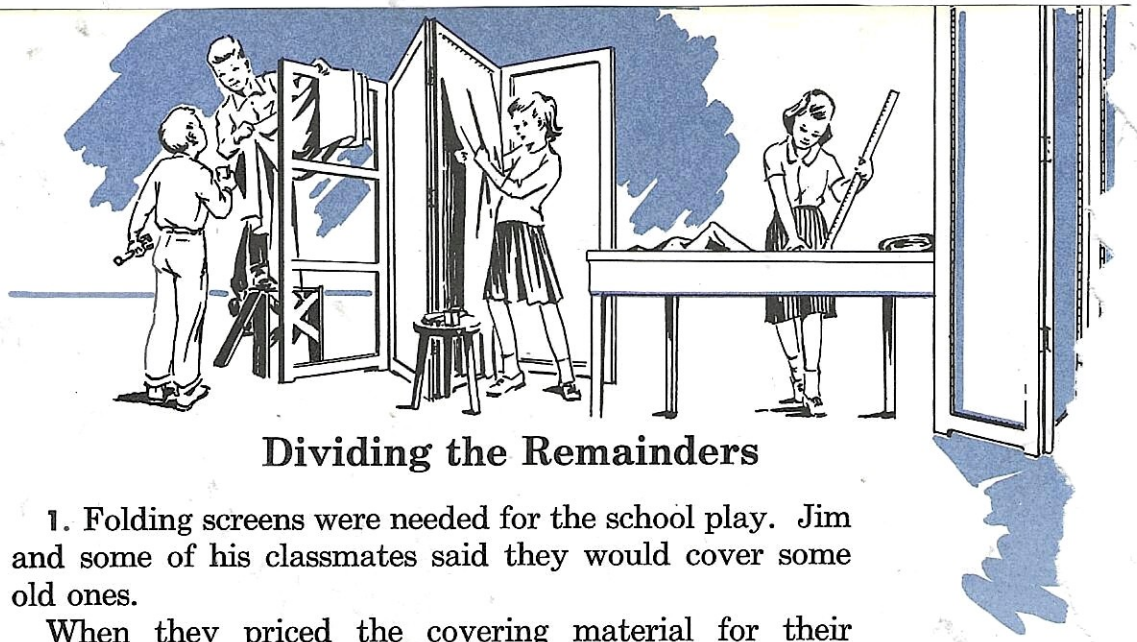
The answer is 341 lengths, 4 ft. remainder.

6.
$$\begin{array}{r} 108\text{ R }2 \\ 8\overline{)866} \\ 8 \\ \hline 66 \\ 64 \\ \hline 2 \end{array}$$

In an exercise like this, you may write the answer as shown.

Find the answers in these division exercises:

	a.	b.	c.	d.	e.	f.
7.	$6\overline{)46}$	$7\overline{)88}$	$12\overline{)114}$	$8\overline{)494}$	$24\overline{)2048}$	$48\overline{)998}$
8.	$9\overline{)825}$	$10\overline{)406}$	$25\overline{)768}$	$6\overline{)139}$	$13\overline{)742}$	$55\overline{)3025}$



Dividing the Remainders

1. Folding screens were needed for the school play. Jim and some of his classmates said they would cover some old ones.

When they priced the covering material for their screens, the boys and girls found that all the prices given them were by the yard. They needed 25 ft. 6 in. of the material. Since there are 36 in. in a yard, they had to divide the number of inches there are in 25 ft. 6 in. by 36.

$$\begin{array}{r} 8 \\ 36\overline{)306} \\ \underline{288} \\ 18 \end{array}$$

There's a remainder of 18 in. that must be turned into yards. It is a fractional part of a yard. Why? 18 in. is $\frac{18}{36}$ of a yard.

Which idea of a fraction is this? Is it 18 parts out of 36 parts of a whole, or is it 18 things compared with a group of 36 things? Could it be either idea?

Why is 18 in. equal to $\frac{1}{2}$ yard?

Do you remember how to reduce $\frac{18}{36}$ to its lowest terms?

$$\begin{array}{r} 8\frac{1}{2} \\ 36\overline{)306} \\ \underline{288} \\ 18 \end{array}$$

$8\frac{1}{2}$ is the quotient. They will need $8\frac{1}{2}$ yd.

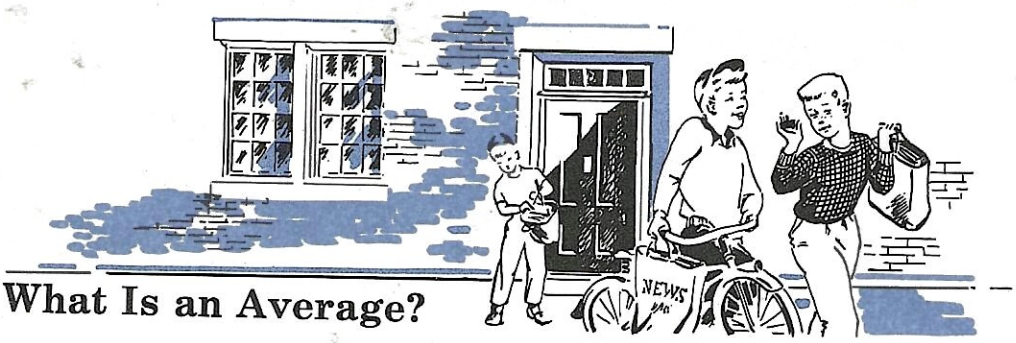
2. Find the quotients:

$8\overline{)86}$ $18\overline{)1086}$ $36\overline{)1024}$ $15\overline{)963}$ $20\overline{)808}$ $18\overline{)256}$

3. If the boys and girls needed 42 ft. 18 in. of material, what would it cost them at \$1.50 a yard?

4. Divide (find fractional parts in quotient):

$16\overline{)324}$ $48\overline{)6948}$ $24\overline{)6248}$ $12\overline{)1854}$ $60\overline{)732}$



What Is an Average?

Do you know what it means to find an average? An average is a number that lies between the largest and smallest of a group of numbers. It is used when you wish to use one number which accurately describes a group of numbers.

For example, while Frank and Bud are on vacation, Jim walks 6 blocks when he delivers Frank's papers and 7 blocks when delivering Bud's papers. The total number of blocks for the 3 routes is 18.

$$\begin{array}{r} 5 \\ 6 \\ 7 \\ \hline 18 \end{array}$$

Since there are 3 routes, you divide the total by 3 to find the average. $18 \div 3 = 6$. The average number of blocks per paper route is 6.

$$\begin{array}{r} 62 \\ 59 \\ 53 \\ \hline 174 \end{array}$$

Jim has 62 customers on his route. Frank has 59, and Bud has only 53. What is the average number of papers delivered on the 3 routes?

$3)174 = 58$ papers, the average for the 3 routes.

Notice that none of the routes took exactly 58 papers, but 58 is the best single number to describe all of them.

1. Harry also delivers papers. He has 33 customers on Ellsworth Street. He has 36 on Sleight Street. If he also has 30 on Wright Street, what is the average number of customers that Harry has per street?

2. There were 128 pages in the Sunday paper on the first Sunday in January, 156 on the 2nd Sunday, 142 on the third Sunday, and 162 pages on the 4th Sunday. What was the average number of pages?

Finding Averages

1. If the rail distance between New York and San Francisco is 3,243 miles, and the scheduled time of a train running the distance is 69 hours, what is the average speed of the train?

2. The air distance between the two cities in Ex. 1 is approximately 2,568 miles. A commercial airline can fly this distance non-stop in approximately 8 hours. What is its average speed?

3. The actual elapsed time for a flight between New York and San Francisco is 12 hours when a refueling stop is made in Chicago. What is the average speed for such a flight?

4. A jet plane recently flew the 3,243 miles in approximately 4 hours. About what was its average speed?

5. The temperature for the week in Jim's city at 4 P.M. was 78, 83, 91, 96, 73, 84, and 90. What was the average daily temperature?

6. A sixth-grade boy went to school 200, 160, 185, 200, and 165 days respectively for each of his previous years in grade school. What was his average yearly attendance?

7. Francis had to carry 16 lb. of chicken feed, 10 lb. of hay, and 60 lb. of seed from a truck to the barn. What was the average weight for the three loads?

8. Nathan earned $1.00 one week and $3.36 the next. What were his average weekly earnings?

9. A boys' club had 10 members. 3 were 10 years old, 6 were 14 years old, and 1 was 8 years old. What was the average age of the members?

Problems and Exercises

1. Pete, Ricky, and Art received $5.18 from the scrapman for iron they collected. How many pounds of iron did they have if the rate was 14¢ a pound?

2. The lumber man offered a total of $10.50 to 6 boys if they would help him load a truck. How much did each boy get if the amount was divided evenly?

3. A football team gained 4,418 yards in one season. If they played 12 games, how many yards did they average per game?

4. Fred and Paul deliver groceries. If they receive $21.28 for making 38 trips, how much do they receive for each trip on the average?

5. Ruth's teacher told her she had been in school 1,200 days. Ruth knew there were about 20 school days in a month. How many months had Ruth gone to school?

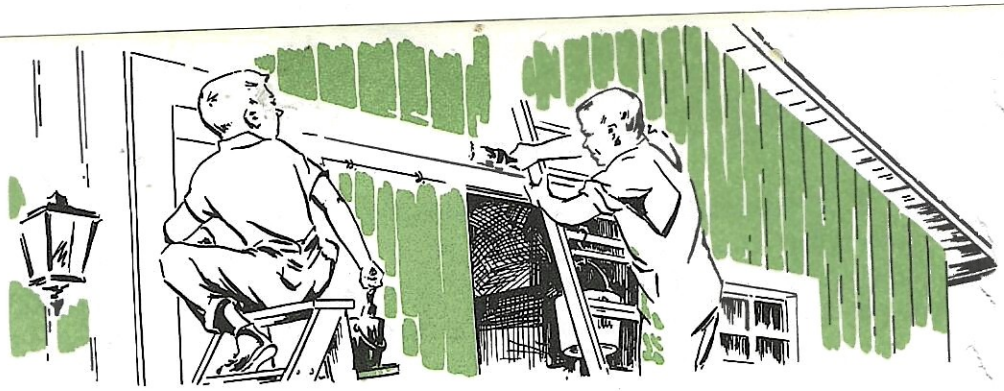
6. How many 16-quart crates would it take to hold 2,048 quarts of strawberries?

7. If the milk plant bottles 976 quarts of milk in an 8-hour day, how many quarts does it bottle per hour?

8. There were 1,968 boxes of canned food in the delivery truck. These were to be divided equally among 8 stores in a town. How many boxes should be delivered to each store?

Divide:

	a.	b.	c.	d.	e.
9.	$16\overline{)2784}$	$25\overline{)3975}$	$64\overline{)448}$	$41\overline{)1704}$	$72\overline{)9864}$
10.	$21\overline{)4494}$	$33\overline{)4192}$	$20\overline{)900}$	$60\overline{)4440}$	$96\overline{)10272}$
11.	$8\overline{)896}$	$12\overline{)960}$	$16\overline{)1792}$	$25\overline{)3650}$	$48\overline{)16446}$
12.	$72\overline{)360}$	$72\overline{)7848}$	$64\overline{)16384}$	$24\overline{)576}$	$64\overline{)40960}$
13.	$25\overline{)32000}$	$14\overline{)20762}$	$16\overline{)2048}$	$25\overline{)6250}$	$10\overline{)10210}$



How to Figure the Cost of Painting

Bill's father asked him to buy enough paint to give the barn one coat. The paint salesman said that each gallon should cover 450 square feet. How many gallons of paint should he buy if there are 9,450 square feet of surface to be painted?

$450\overline{)9450}$

Using the general method Bill must estimate the first quotient figure. He thought: My way of estimating is to find how many 4's there are in 9. There are 2, but where do I place the 2 as a quotient figure?

$$\begin{array}{r} 21 \\ 450\overline{)9450} \\ 900 \\ \hline 450 \\ 450 \\ \hline \end{array}$$

Why didn't Bill think of using: How many 450's are there in 945?

Which would you use? Why?

Would you have thought this way: Is my quotient a two-place number as large as 10 ($10 \times 450 = ?$), or as large as 20 ($20 \times 450 = ?$)?

Sometimes you have to multiply in order to find the place for the first quotient figure. Of course, the right place for the first quotient figure is above the last place of the first usable part.

1. The gym floor in your school needs to be revarnished. How many gallons of varnish are necessary to cover the 7,500 square feet of gym floor, if one gallon covers 250 square feet?

2. Divide:

$729\overline{)5832}$ $144\overline{)5984}$ $513\overline{)7695}$ $512\overline{)8192}$ $144\overline{)1728}$

Practice Problems and Exercises

1. John and Henry began their garden on May 15 and worked 17 weeks until September 15. During that time, they each spent a total of 125 hours in their garden. How much time did they spend per week?

2. After deducting the cost of the seeds from the amount they received, the 2 boys had $265.20 in all. What did each boy earn per week if the amount was divided equally?

3. How much did each boy earn per hour? Is that a good wage? Should they work in a factory next summer for $.90 per hour?

4. They found that they had worked 30 hours with their tomato plants, and had received $87.50 more for tomatoes than they had spent. Did they earn at the same rate on their tomatoes as they did on the total garden produce?

5. They worked 63 hours on their cabbages and collected $28.77 more for them than they spent. Which gave them the larger hourly earnings, the cabbages or the tomatoes?

Find the quotient:

	a.	b.	c.	d.	e.
6.	$10\overline{)360}$	$8\overline{)408}$	$20\overline{)440}$	$15\overline{)230}$	$7\overline{)343}$
7.	$25\overline{)775}$	$30\overline{)660}$	$4\overline{)30056}$	$5\overline{)21040}$	$8\overline{)61080}$
8.	$6\overline{)37002}$	$50\overline{)40400}$	$24\overline{)16416}$	$40\overline{)28080}$	$75\overline{)70500}$
9.	$68\overline{)15436}$	$37\overline{)13368}$	$91\overline{)28756}$	$84\overline{)45024}$	$300\overline{)28200}$
10.	$450\overline{)28900}$	$205\overline{)15170}$	$525\overline{)19950}$	$600\overline{)29400}$	$12\overline{)204}$
11.	$850\overline{)49300}$	$12\overline{)765}$	$23\overline{)349}$	$46\overline{)377}$	$37\overline{)493}$
12.	$19\overline{)4631}$	$55\overline{)2974}$	$31\overline{)5724}$	$68\overline{)3464}$	$107\overline{)34603}$

Review—Finding Fractional Parts in Quotients

When you are measuring paint, boards, or many other things, you usually have remainders. If the remainders are changed into fractions in the quotients, it is sometimes easier to use this answer in your work.

1. Divide 4,689 things into 4 equal parts.

$$
\begin{array}{r}
1172\frac{1}{4} \\
4\overline{)4689} \\
\underline{4} \\
6 \\
\underline{4} \\
28 \\
\underline{28} \\
9 \\
\underline{8} \\
1
\end{array}
$$

When you solve this problem, you will find that only 4688 whole things are usable; there is one left over. This is called a remainder.

When you divide 1 into 4 equal parts, you get $\frac{1}{4}$. Thus, each of the 4 equal parts will be $1172\frac{1}{4}$.

2. Three farmers worked a certain large wheat field together and were to divide the yield among them. The field yielded 1,372 bushels. How many bushels should each receive?

3. A bakery made 1,463 doughnuts one Saturday. They planned to sell them in packages of 6 doughnuts each. How many packages would they have? Will there be a fractional part of a package? If so, how many doughnuts in this package?

Find the quotient:

	a.	b.	c.	d.	e.
4.	$7\overline{)884}$	$6\overline{)345}$	$8\overline{)1082}$	$5\overline{)943}$	$8\overline{)676}$
5.	$12\overline{)4612}$	$14\overline{)4531}$	$22\overline{)8767}$	$24\overline{)10254}$	$64\overline{)8196}$

6. If 3 ft. is divided into 4 equal parts, each part equals $\frac{1}{4}$ of 3 ft. or $\frac{3}{4}$ ft. Why?

7. What are the fractional parts in these? (Label your answers.)

$4\overline{)66 \text{ in.}}$ 4 in.$\overline{)66 \text{ in.}}$ 12 in.$\overline{)42 \text{ in.}}$ $18\overline{)96 \text{ yd.}}$

Practice for Skill

Find the quotients:

	a.	b.	c.	d.	e.	f.
1.	$4\overline{)72}$	$4\overline{)54}$	$2\overline{)34}$	$3\overline{)42}$	$5\overline{)60}$	$5\overline{)75}$
2.	$15\overline{)120}$	$13\overline{)117}$	$16\overline{)192}$	$19\overline{)437}$	$18\overline{)828}$	$13\overline{)195}$
3.	$34\overline{)748}$	$98\overline{)9702}$	$43\overline{)2408}$	$72\overline{)2736}$	$24\overline{)936}$	$56\overline{)2408}$

Divide:

4.	$24\overline{)46}$	$48\overline{)456}$	$72\overline{)594}$	$20\overline{)170}$	$64\overline{)400}$	$36\overline{)333}$

	a.	b.	c.	d.	e.
5.	$387\overline{)16641}$	$473\overline{)26015}$	$292\overline{)10220}$	$288\overline{)11520}$	$520\overline{)32240}$

	a.	b.	c.	d.	e.	f.
6.	$6\overline{)90}$	$4\overline{)72}$	$3\overline{)54}$	$2\overline{)90}$	$6\overline{)84}$	$4\overline{)56}$
7.	$15\overline{)450}$	$16\overline{)160}$	$17\overline{)153}$	$19\overline{)988}$	$14\overline{)280}$	$13\overline{)689}$

Find the remainders:

8.	$43\overline{)8764}$	$81\overline{)4320}$	$76\overline{)2187}$	$89\overline{)6432}$	$65\overline{)9741}$	$59\overline{)6872}$

Divide:

9.	$24\overline{)100}$	$40\overline{)170}$	$36\overline{)909}$	$42\overline{)258}$	$55\overline{)451}$	$64\overline{)712}$

	a.	b.	c.	d.	e.
10.	$643\overline{)52083}$	$894\overline{)25926}$	$432\overline{)27216}$	$146\overline{)13140}$	$289\overline{)18785}$

	a.	b.	c.	d.	e.	f.
11.	$4\overline{)48}$	$3\overline{)93}$	$2\overline{)26}$	$5\overline{)55}$	$2\overline{)46}$	$3\overline{)36}$

Find the remainders:

12.	$13\overline{)6248}$	$11\overline{)6429}$	$17\overline{)2304}$	$16\overline{)1880}$	$19\overline{)2557}$	$22\overline{)8069}$

Divide:

13.	$12\overline{)27}$	$42\overline{)266}$	$27\overline{)153}$	$28\overline{)77}$	$56\overline{)266}$	$72\overline{)480}$

Unit Review

1. The sixth-grade class wanted to buy an aquarium for their classroom. They didn't have any money so they decided to bake and sell cookies.

The class was divided into groups of 8 children each to do the work. To make it more fun, Margaret said, "Let's have a contest and see which group can bake and sell the most cookies."

Linda's group won, selling 128 dozen. What was the average number sold by each pupil in her group?

Carol's group sold the least number of cookies. If their total was 104 dozen, what was the average number sold per pupil?

Dick's group sold 113 dozen, and Rosemary's group sold 119 dozen. What was the average number sold by each group?

2. Wilma, Helen, Jean, Margaret, and Ginny volunteered to write the announcements for their class play. Wilma wrote 37; Helen, 28; Jean, 32; Margaret, 29; and Ginny wrote 44. What was the average number written by each girl?

3. Ed's cows ate 182 bales of hay during the week. What was the average number of bales eaten per day?

4. Jack's football team scored 243 points during its nine-game season. What was the team's average score per game?

5. Bill mowed lawns last summer. One week he earned the following: \$1.25, \$1.80, \$.50, \$.90, \$2.00, \$1.75, \$1.95 and \$1.80. How much did he average per lawn?

6. Find only the first quotient figure:

$6\overline{)418}$ $48\overline{)4975}$ $18\overline{)768}$ $9\overline{)4036}$ $26\overline{)8875}$ $260\overline{)7964}$

Divide:

7. $12\overline{)144}$ $16\overline{)256}$ $20\overline{)880}$ $25\overline{)1125}$ $19\overline{)361}$

8. $8\overline{)4008}$ $17\overline{)361}$ $21\overline{)441}$ $48\overline{)2304}$ $100\overline{)1500}$

Unit Test

1. How many folding screens can be covered with a 225-foot bolt of cloth if 6 ft. are needed for each screen?

2. Pencils are cheaper if they can be bought by the gross. If there are 24 children in a room, how many pencils will there be for each child from a gross? (There are 12 dozen in a gross.)

3. Mr. Bartlett's class wanted a jungle gym for their playground. They decided to have a carnival to raise the money. How many tickets would they have to sell at 25¢ each to raise $137.50?

4. If your family's car will average 21 mi. for each gallon of gas, how many gallons will it take to go 609 miles?

5. If a tourist could average 38 mi. per hour, how long would it take him to make a trip of 3,610 mi.?

6. If the milk bill for the Elliot School shows that they received 2,500 bottles of milk for the 20 school days in one month, how many bottles did they receive each day?

7. Scout troop #248 earned $128.64 in their paper drive. The money is to be divided equally among the 8 patrols. How much should each patrol receive?

8. There are 3,444 books to be shelved. How many shelves are needed if each shelf is to hold 28 books?

9. A farmer had one 20 A. field of corn that averaged 85 bu. to the acre, and another 20 A. that averaged 95 bu. What was the average yield per acre for his total crop?

Divide:

10. $4\overline{)84}$ $4\overline{)64}$ $9\overline{)78}$ $13\overline{)156}$ $59\overline{)2001}$

11. Find the remainder of $22\overline{)8163}$.

12. When you divide 5 ft. into 8 equal parts, what is your answer? Why?

13. Divide, giving fractional values in the quotient:

$28\overline{)172}$ $139\overline{)11815}$

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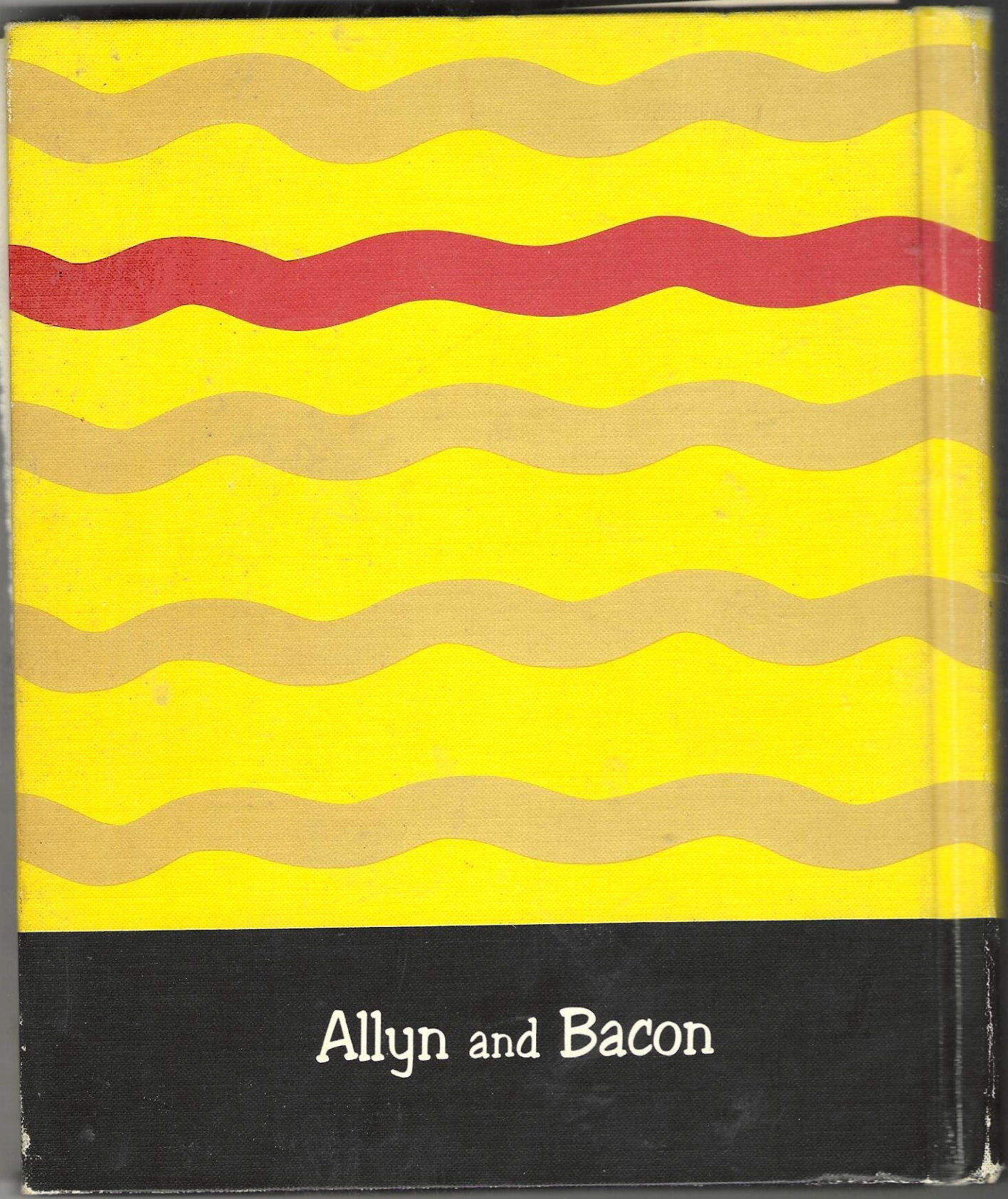

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